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Multi-Objective Optimization Application in Power Systems

by

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Abstract

For hundreds of years, electricity supply industries have been in the hands of monopoly utilities, until engineers conceived the management of this industry as a challenging multi-objective optimization problem. Multiple objectives need to interact in divergent or competing interests to win the competition and to deliver the electrical energy and keep the lights on. The most highlighted objectives are the economic dispatch of power for satisfying consumers with lower power bill costs, the minimum loss in transmission lines, the environmental emission reduction, and the reliability and stability of the voltage and generated power. The consideration of mentioned goals push for a tighter coordination of planning and operation scheduling in a power system and raises interest in using multi-objective optimization methods in power system applications.

The Optimal Power Flow (OPF) is used to determine an optimal operating condition for power systems while considering the limitations and system constraints such as: generator capability, line capacity, bus voltages, and power flow balances.

Among many optimization methods, the Teaching-Learning Based Optimization (TLBO) algorithm has gained wide acceptance in different areas of science and engineering. In this study, the Teaching-Learning-Based Optimization algorithm is used to optimize the Optimal Power Flow problem considering total fuel cost of generation, emission, voltage deviation and, active power transmission losses in single and multi-objective cases. The method has been applied on the IEEE 30-bus and IEEE 57-bus test systems for several OPF objectives including minimization of fuel cost, emission, voltage deviation, and power losses, and the results reflect the effectiveness of the method.

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Chapter 1

1. Introduction

Electrical energy is considered to be the most popular form of energy globally, because it can be transported at high efficiency and at reasonable prices [1]. For about hundreds of years, the electricity supply industries were in the hands of monopoly utilities, until engineers conceived the management of this industry as a challenging optimization problem. By increasing the size, complexity, and scope of these optimization problems, professional and powerful computer programs were designed and deployed to refine the most efficient algorithms for the planning and the operation of power systems. By introducing competition to the electricity supply industry, a single organization is no longer in charge. Multiple objectives need to interact in divergent or competing interests to win the competition and to deliver the electrical energy and keep the light on [2]. The most highlighted objectives are the economic dispatch of power for satisfying consumers with lower power bill costs, the reliability and stability of generated power, the minimum loss in transmission lines, and the security constraints in a power system.

The power transmitted through transformers and transmission lines is restricted to the given limits, which arise due to thermal, voltage, or stability considerations. The Optimal Power Flow (OPF) is used to determine an optimal operating condition for power systems while considering the limitations and system constraints such as generator capability, line capacity, bus voltages, and power flow balances. Increasing the power quality and reliability, using the available reserves are other possible target goals in solving an OPF problem. In this study, the OPF is formulated as a multi-objective problem of minimizing fuel cost, emission, transmission losses, voltage deviation, etcetera.

Security Constrained Optimal Power Flow (SCOPF) is defined as an effective strategy to enforce the reliability criteria for the secure operating of the system. The achieved operating point ensures that if any possible contingency happens, the post-contingency states remain secure [3].

The consideration of mentioned goals push for the tighter coordination of planning and operation scheduling in a power system and raises interest in using multi-objective optimization methods in power system applications.

Among the many optimization methods, the Teaching-Learning-Based Optimization (TLBO) algorithm has gained wide acceptance among optimization researchers [4]. In this study, the TLBO algorithm is used to optimize the OPF problem in several single and multi-objective cases. The method has been applied on the IEEE 30-bus and IEEE 57-bus test systems for several OPF objectives such as minimization of fuel cost, emission, voltage deviation, and power losses.

1.1 Research Objectives

The focus of this research is to present the application of multi-objective optimization methods in power system problems. Different objectives of the OPF problem have been introduced with constraints and limitations.

The TLBO method is used to optimize different OPF objectives, in a single or multi-objective optimization problem, that reflects the performance of power systems. Furthermore, the TLBO algorithm has been applied to the optimization problem as an effective method.

The principal goals of this research are summarized as follows:

- To study applications of optimization techniques in power systems.
- To study the constrained optimization methods and applications for Economic Dispatch (ED) and Optimal Power Flow(OPF).
- To research and explore the multi-objective optimization methodologies in power system operations.
- To study security constraints in power system problems.

- To study, analyze, and visualize the operation and planning of a power system using MATLAB, as a powerful computer programming software, and the PowerWorld simulator
- To study the effectiveness of the Teaching-Learning-Based Optimization (TLBO) algorithm in the Optimal Power Flow problem for several single and multi-objective cases.

1.2 Thesis Organization

This thesis consists of 6 chapters. Chapter 2 provides a review of optimization techniques. A comprehensive model of optimization problems and the classification of methods are discussed. Local and global algorithms are introduced. In addition, the application of MATLAB optimization toolbox is briefly introduced to solve several case studies and illustrates the effectiveness of algorithms for general optimization problems.

Chapter 3 deals with theories and principles behind three major types of problems commonly referred to in power systems literature as power flow, Economic Dispatch (ED) and Optimal Power Flow (OPF). Different formulations of ED are applied to achieve the least-cost generation dispatch, considering and neglecting transmission loss and generation limits. The main principles of the optimal power flow problem and Security Constraint Optimal Power Flow (SCOPF) are proposed. For each type of problem, case studies and examples are provided to illustrate the concepts. MATLAB and Power World simulator are used to analyze and simulate the 7-bus and 37-bus power systems.

Chapter 4 presents the concepts and classification of single and multi-objective optimization problems. The Teaching-Learning-Base Optimization (TLBO) algorithm is introduced and the method is implemented on a constraint optimization problem.

Chapter 5 is concerned with optimizing 4 different objectives of the OPF problem: fuel cost, environmental emission, voltage deviation, and transmission losses. The constrained OPF problem is formulated and optimized in different scenarios, as a single objective or multi-objective optimization problem, and the results are compared.

Chapter 6 highlights the key contribution of the research presented in this thesis along with suggestions for future works.

Chapter 2

2. Review of Optimization Techniques

2.0 Introduction

“For since the fabric of the universe is most perfect, and is the work of a most wise Creator, nothing whatsoever takes place in the universe in which some form of maximum or minimum does not appear.” Leonhard Euler ¹

Many researchers have shown that humankind has the instinctive desire to make the most beneficial decisions and choose the optimal solution between possible options, for the purpose of heightening the quality of life. From the early ages of civilization until now, there have been ongoing struggles toward developing the core principles of optimization techniques to make the best choices among any inferior situation.

There have been extensive efforts toward this aim by dedicated scientists, engineers, mathematicians, and philosophers, which have resulted in the development of methods, tools, and intelligent softwares we use today. It is imperative to discuss the major achievements in this area.

This chapter draws the focus to major optimization techniques, and briefly explains the mathematical theory and foundations behind them.

¹ Leonhard Euler, Introduction to De Curvis Elasticis, Additamentum I to his Methodus Inveniendi Lineas Curvas Maximi Minimive Proprietate Gaudentes 1744; translated on page 10- 11, "Leonhard Euler's Elastic Curves", Old father et al 1933.

2.1 Optimization

Optimization is the process of maximizing or minimizing the desired objective function while satisfying the problem constraints. Scientists have shown that an abundant number of processes and structures in nature follow an optimum system status. For example, one of nature's optimization processes is genetic mutation for survival. As well, the crystalline structure of metal and alloys is defined by unit cells that are formed by atoms in the minimum energy positions. The sphere of a liquid droplet in zero gravity is the geometric form of the minimum surface area for a given volume. The architecture of a honeycomb is one of the most compact packaging structures. Many experts and managers use optimum solutions as a key in their organizations and businesses to strive for excellence. This attitude helps them to achieve consumer satisfaction by identifying optimum solutions from different feasible scenarios. A production factory would benefit hugely from small savings in a mass-produced part. By minimizing the weight of a vehicle, fuel consumption can be affected significantly and this would increase the efficiency. A company needs to consider the optimum number of labor resources for a maximized profit. An automated production line could minimize production time per item by optimizing the process cycle and could ultimately maximize efficiency and benefit [1].

The challenge of increasing the efficiency of known optimization techniques has been an ongoing struggle over the decades. Today, applications of optimization are found in all aspects of our lives.

2.1.1 The Standard Form of the Optimization Problem

It is important to present the standard form of optimization problems before reviewing the optimization methods. Optimizing a scenario in an engineering problem can be defined as minimizing or maximizing a function with respect to inequality and equality constraints. Considering real cases, the optimization model can be formed by objectives and constraint functions, which may be linear or nonlinear, convex or non-convex, explicit or implicit functions.

The standard form for a single-objective, non-linear, constrained optimization problem is provided in Equation 2.1.

$$\begin{aligned}
 & \text{Minimize} && f(x) && (2.1) \\
 & \text{Subject to} && g_i(x) \leq 0 && i = 1, \dots, m \\
 & && h_j(x) = 0 && j = 1, \dots, \ell \\
 & && x_i^L \leq x \leq x_i^U
 \end{aligned}$$

where $x = (x_1, x_2, \dots, x_n)^T$ is a column vector of n real-valued design variables, which are modified to obtain the optimal solution. In Equation 2.1 $f(x)$ represents the objective (or goal) function, $g_i(x)$ is an inequality constraint and $h_j(x)$ stands for an equality constraint function. The searchable design region is determined by the upper and lower bounds, x_i^L and x_i^U , of the design variables, referred to as the side constraints [2].

Figure 2.1 represents a nonlinear, constraint optimization problem for $n=2$ design variables, where constant curves for objective function contours are drawn. The optimum is expressed by the highest contour that passes through the feasible region. In this example, $\Omega = \{x: g(x) \leq 0, h(x) = 0, x_i^L \leq x_i \leq x_i^U\}$. Ω , a subset of R^n , is called the feasible region. Vectors x_i^L , x_i^U represent the explicit lower and upper bounds on the design variables, respectively. The optimization problem is presented in Equation 2.2.

$$\begin{aligned}
 & \text{Minimize } f(x) && (2.2) \\
 & \text{Subject to } x \in \Omega
 \end{aligned}$$

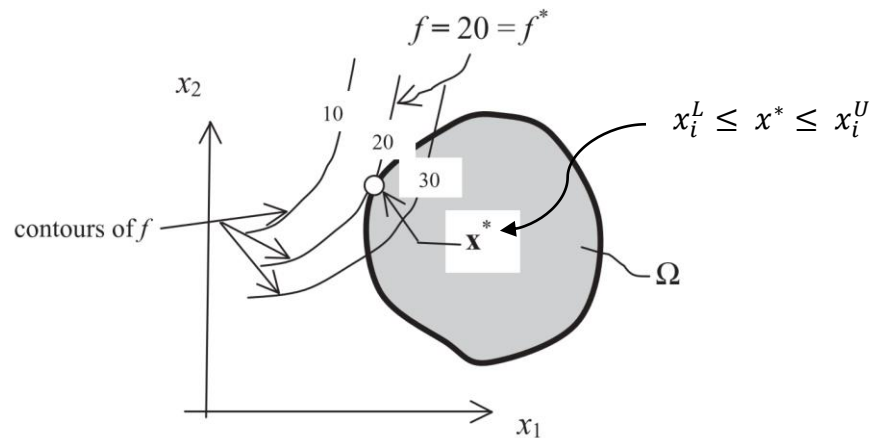


Figure 2.1: Graphical Representation of Nonlinear Programming Problem in x-Space [5]

Figure 2.2 represents the global and local optima of a two-dimensional function. Ashok D.B [5] defines the mathematical definition of global and local optima in his book according to the following definitions [5].

Local Maximum: By definition, a (local) maximum $x^* \in \mathbb{X}$ of an objective function $f: \mathbb{X} \rightarrow \mathbb{R}^n$ is an input element with $f(x^*) \geq f(x)$ for all x neighboring x^* . If $\mathbb{X} \subseteq \mathbb{R}^n, \forall x^* \exists \varepsilon > 0, f(x^*) \geq f(x), \forall x \in \mathbb{X}, |x - x^*| < \varepsilon$.

Local Minimum: A (local) minimum $x^* \in \mathbb{X}$ of an objective function $f: \mathbb{X} \rightarrow \mathbb{R}^n$ is an input element with $f(x^*) \leq f(x)$ for all x neighboring x^* . If $\mathbb{X} \subseteq \mathbb{R}^n, \forall x^* \exists \varepsilon > 0 : f(x^*) \leq f(x), \forall x \in \mathbb{X}, |x - x^*| < \varepsilon$.

Local Optimum: A local optimum $x^* \in \mathbb{X}$ of an objective function. If $f: \mathbb{X} \rightarrow \mathbb{R}^n$ is either a local maximum or a local minimum.

Global Maximum: A global maximum $x^* \in \mathbb{X}$ of an objective function. If $f: \mathbb{X} \rightarrow \mathbb{R}^n$ is an input element where $\forall x \in \mathbb{X}, f(x^*) \geq f(x)$.

Global Minimum: A global minimum $x^* \in \mathbb{X}$ of an objective function If $f: \mathbb{X} \rightarrow \mathbb{R}^n$ is an input element where $\forall x \in \mathbb{X}, f(x^*) \leq f(x)$.

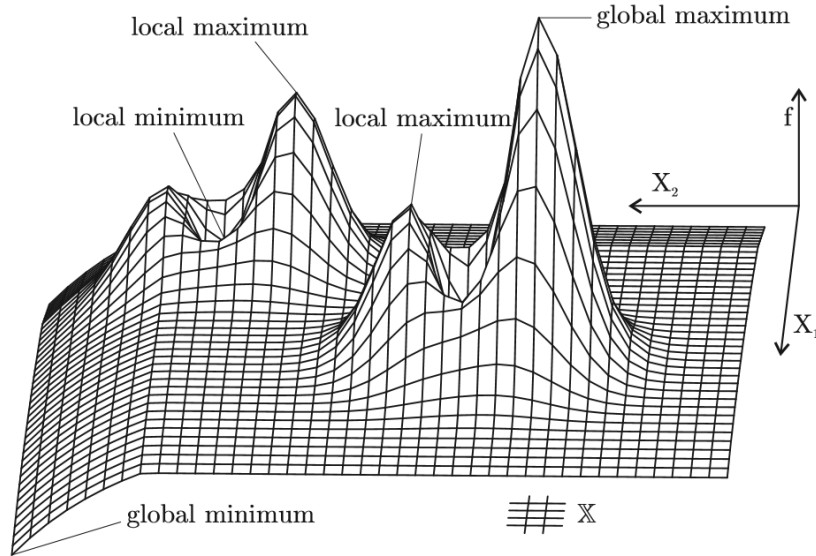


Figure 2.2: Global and Local Optimal of a Two-Dimensional Function [5]

Global algorithms are highly efficient for solving problems with integer and/or discrete variables. In a discrete or integer optimization problem, some or all of the design variables are

restricted to be an integer or discrete value. Generally, implicit functions appear in this class of problems.

The side constraints in an optimization problem are commonly implemented directly on the algorithm rather than considering equality or inequality constraints. By definition, solving a constrained optimization problem is the process of optimizing an objective function with respect to one or more equality and/or inequality constraints, with or without side constraints. An equality constraint can either be violated or satisfied, while an inequality constraint can be violated, active, or satisfied. An active inequality constraint is one for which $g_j(x) = 0$. An unconstrained optimization problem may have side constraints, while it will not be affected by any equality or inequality constraints [5].

2.2 An Overview of Classifications of Optimization Techniques

Optimization techniques are used to find the best solution to the problem presented in Equation 2.1. The approach is to form the optimum objective function by choosing the best combination of design variables in the searchable region while satisfying all the equality, inequality and side constraints. Referring to local or relative optima for some cases, more than one optimum may exist.

An important step in optimizing a problem is classifying the optimization model. This section presents an overview of the optimization model classification for the various optimization problem types [6].

2.2.1 Continuous Optimization versus Discrete Optimization

In an optimization process, the appropriate algorithm can be identified based on the nature of the design variables, which can be either continuous or discrete values. In discrete optimization problems, the variables are only allowed to take on values from a set of discrete elements (a subset of integers). In a continuous optimization problem, any real value can be implemented in the model.

According to the property of smoothness in a continuous function, it is feasible to derive information about the neighborhoods of a point, based on the provided values of the objective

function and constraints in that specified point. Many discrete optimization problems make use of generating continuous domain subsets to simplify the computation process.

2.2.2 Unconstrained Optimization versus Constrained Optimization

Optimization problems can be categorized based on the existence of constraints in the problem structure. An unconstrained optimization problem does not include any active/violated constraints in its model. In a constrained optimization problem, the objective function is supposed to be optimized with respect to one or more equality and/or inequality constraints. Also, in some applications, a constrained problem is reformulated by using a penalty factor in the objective function and an unconstrained optimization problem will arise in this case.

2.2.3 Single Objective or Multi Objectives Functions

The number of objective functions in an optimization problem can define another category of optimization methods. Single-objective optimization corresponds to the minimum or maximum value of a single objective function. The research of multi-objective optimization methods received impetus in many practical engineering fields, where the problem needs to achieve a compromise between two or more conflicting objective functions and finds the best set of values for each one of them.

2.2.4 Deterministic Optimization versus Stochastic Optimization

Deterministic optimization methods use accurate known data, while in a stochastic optimization process, the set of information is not actual (as a result of measurement error or representing uncertain data about a future process).

The deterministic solutions “become computationally impracticable for problems of realistic size, either because the model grows too large, or because the solution procedures are too lengthy, or both, and heuristics provide the only viable scheduling techniques for large projects” [7]. In uncertain optimization methods, the uncertainty is incorporated into the mathematical formula of the model.

Stochastic optimization models make use of probabilistic distribution functions through the feasible region of the problem (these functions are either available or can be estimated). The

main goal is to approach a strategy that is feasible for all possible data and optimizes the future efficiency of the model. Some references [4] have presented another classification of optimization techniques based on the nature of the equations involved in the objective function. The most famous methods in this category include linear/ nonlinear optimization, geometric optimization, and quadratic optimization.

There are many options for classifying the most common approaches in optimization techniques. In the following section, a brief overview of a broad classification, as either local or global algorithms, is provided. Figures 2.3 and 2.4 represent two different taxonomies of optimization techniques [6], [8].

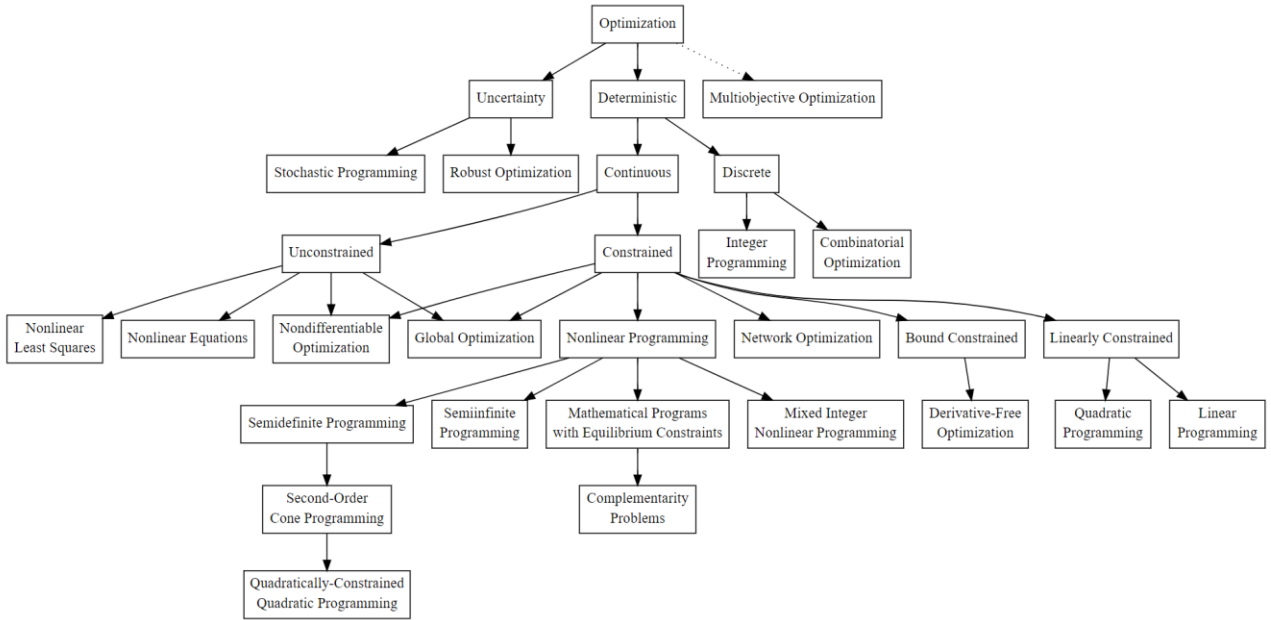


Figure 2.3: Optimization Taxonomy [6]

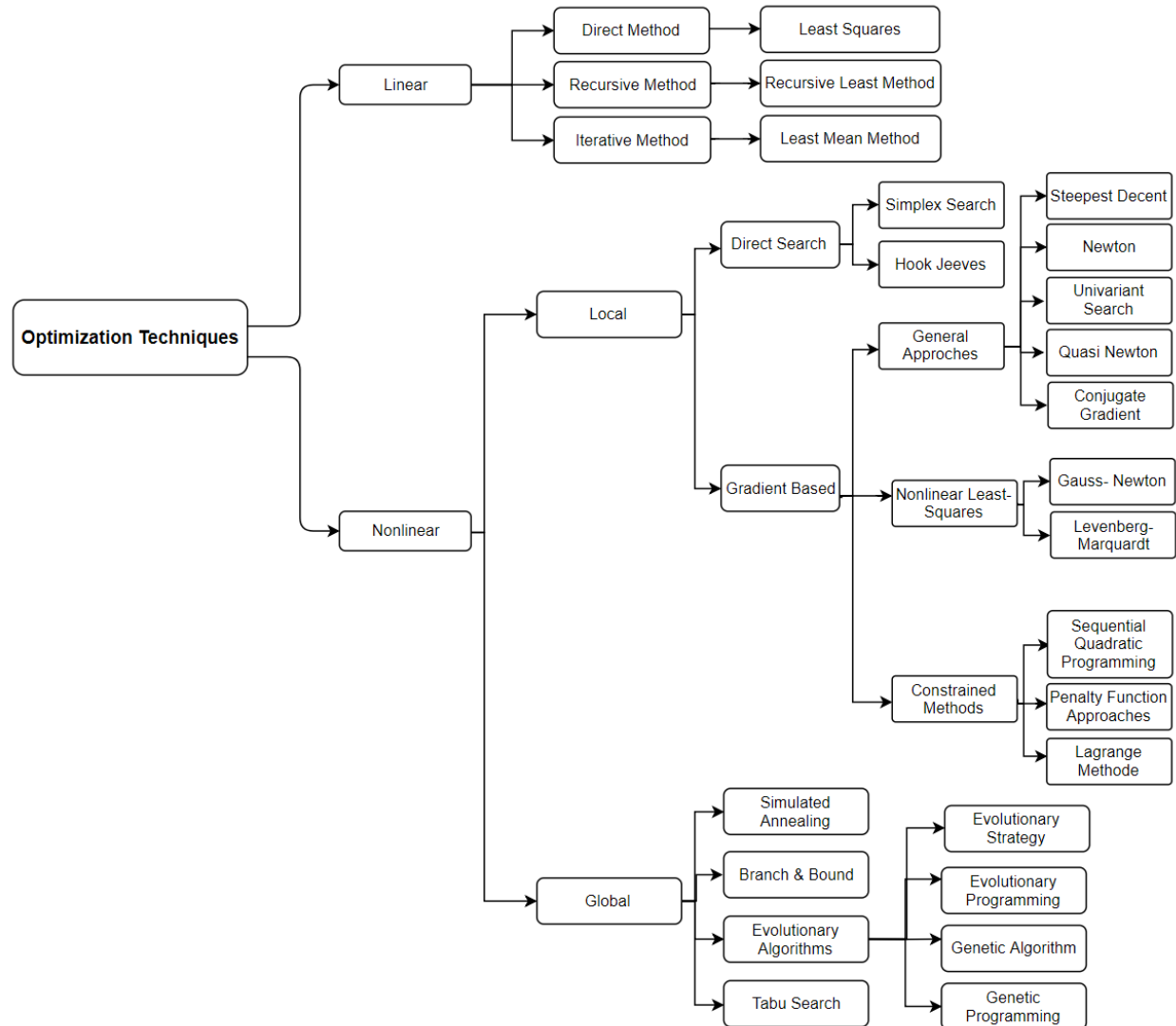


Figure 2.4: Classification Schematic of Optimization Techniques For Engineering Systems [8]

2.2.5 Choice of Method

Several factors should be considered in deciding which particular method to solve a given optimization problem. Some of these factors are [4]:

1. The type of problem to be solved (general nonlinear programming problem, geometric programming problem, etc.)
2. The availability of a ready-made computer program
3. The calendar time required for the development of a program

4. The necessity of derivatives of the functions f and g_j , $j = 1, 2, \dots, m$
5. The available knowledge about the efficiency of the method
6. The accuracy of the solution desired
7. The programming language and quality of coding desired
8. The robustness and dependability of the method in finding the true optimum solution
9. The generality of the program for solving other problems
10. The ease with which the program can be used and its output interpreted [4].

2.3 Local Optimization Algorithms

Local optimization techniques are heuristic methods that are widely used in a variety of engineering problems. The optimization procedure in this classification consists of searching in the feasible criterion among candidate solutions by making local changes in each outcome solution. This strategy is continued until a set of solutions deemed to be optimal is found or a time boundary is met.

Most local optimization techniques make use of gradient information to find the optimal solution. The popularity of these methods is due to their high efficiency, as they can solve problems with a large number of variables while demanding little problem-specific parameter tuning. However, heuristic methods and local search algorithms do not guarantee the best optimum solution but attempt to yield a local optimum in the feasible region. They also have difficulties in solving discrete optimization problems.

Gradient-based algorithms follow a two-step process to reach the optimum solution in most cases. This process can be explained using Figure 2.5 of a blindfolded boy on a hill. The boy is supposed to reach the highest point on the hill (maximizing the objective function) while staying inside the fences (satisfying the constraints and staying in feasible criterion). The design variables are the x and y coordinates of the boy.

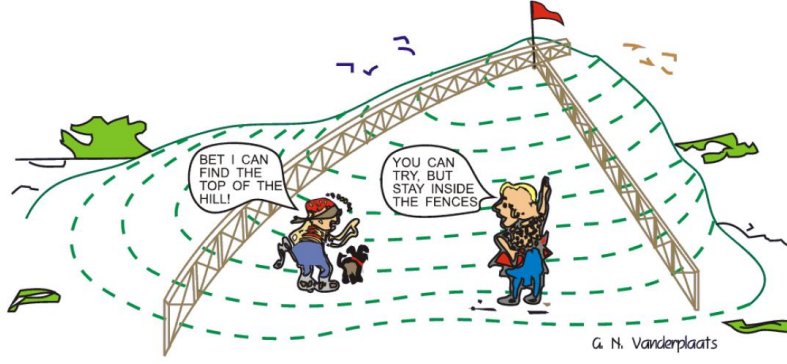


Figure 2.5: Gradient-based Optimization [9]

Based on the gradient optimization technique, the blindfolded boy can use a two-step process in order to reach the top of the hill (a single step in the x-direction and another step in the y-direction), while trying to estimate his next step direction based on information gained from previous steps. His walk in this direction will continue until no further progress can be made, which may include reaching a fence. In this position, he is able to choose a new direction by taking another two steps inside the fences (as a feasible region). Then he can continue the process until he reaches the peak of the hill. This two-step iterative method of searching for the optimum solution can be mathematically summarized according to Equation 2. 3.

$$x^q = x^{q-1} + \alpha^* S^q \quad (2.3)$$

S: Searching direction

α^* : Optimum step size

In this example, the first step is to use gradient information, which determines the searching direction ‘S’ to move. The second step provides the optimum step size, α^* , which represents the number of iterations that should be followed in this direction until no progress can be yielded. Termination of local optimization techniques can be based on different choices, such as time boundaries or the number of iterations. Different search directions can be applied to the optimization problem. A usable direction that could enhance the objective function is desired for an unconstrained optimization problem or a constraint optimization problem without active/violated constraints.

On the other hand, the required search direction for a constraint optimization scenario with violated constraints is desired to overcome the constraint bounds and is called a feasible direction. If the problem includes active constraints and no violated constraint, the search

direction method should be both usable and feasible, which simply means that it should not violate any constraint bounds and it should improve the objective function at the same time.

Considering the one-dimensional search, the best step size could be found through many different methods, including the golden section search, the Fibonacci search, and many variations of polynomial approximations. Then the desired technique can be combined with a specific gradient-based algorithm to perform the required one-dimensional search.

Karush-Kuhn-Tucker (KKT) [5] conditions can be used to determine whether a constrained local optimum has been found by the presented gradient information. Karush-Kuhn-Tucker conditions provide the necessary conditions for a local optimum and can be summarized as:

1. The optimum design point x^* must be feasible.
2. At the optimum design point, the gradient of the Lagrangian must vanish according to Equation 2. 4,

$$\nabla f(x^*) + \sum_{j=1}^m \lambda_j \nabla g_j(x^*) + \sum_{k=1}^p \lambda_{m+k} \nabla h_k(x^*) = 0 \quad (2. 4)$$

where the Lagrange multipliers, $\lambda_j \geq 0$ and λ_{m+k} , are unrestricted in sign.

3. For each inequality constraint $\lambda_j \nabla g_j(x) = 0$, where $j = 1, \dots, m$.

Although the Karush-Kuhn-Tucker conditions are effective in determining the local optimum, their satisfaction cannot guarantee that the global optimum has been achieved [5].

2.3.1 Newton's Method

Newton's algorithm is one of the classical gradient-based optimization solutions. It has been classified as an iterative unconstrained algorithm that is derived from a second-order Taylor series expansion of the objective function around the initial design point x^0 according to Equation 2.5.

$$f(x) \approx f(x^0) + \nabla f(x^0)^T (x - x^0) + \frac{1}{2} (x - x^0)^T H(x^0) (x - x^0) \quad (2.5)$$

where $H(x^0)$ is the Hessian matrix, and contains the second-order gradient information (partial derivatives) of the objective function in a square matrix. Differentiating Equation 2.5 with respect to x and setting the result equal to zero according to the Karush-Kuhn-Tucker conditions, the updated formula in Equation 2.6 for the current design point will result.

$$x = x^0 - H(x^0)^{-1} \nabla f(x^0) \quad (2.6)$$

And for higher dimensions, the above iterative scheme can be generalized in Equation 2.7.

$$x_{n+1} = x_n - \gamma H(x_n)^{-1} \nabla f(x_n) \quad (2.7)$$

Newton's method is modified to include a small step size $\gamma \in (0,1)$.

A presented classical form of the Newton algorithm in equation 2.6 makes use of a fixed step size of $\gamma = 1$ and provides the search direction (descent direction) of $-H(x^0)^{-1} \nabla f(x^0)$.

The method follows a quadratic rate of convergence, requiring a single step to reach the optimum point for any positive definite quadratic function. In a practical approach, Newton's method is adjusted to include a one-dimensional search that enhances both the efficiency and robustness of the method.

The method has a highly desirable quadratic convergence rate; meanwhile, the calculation process associated with finding the second-order gradient information in the Hessian matrix would be highly sophisticated. As a result, most gradient-based methods make use of first-order gradient information to simplify the method.

Figure 2.6 represents a comparison of gradient descent route (green) and Newton's method route (red) for minimizing a function (with small step sizes) where Newton's method uses curvature information to take a more direct (shorter) route.

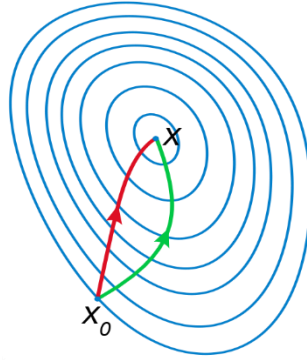


Figure 2.6: Newton's Method Uses Curvature Information to Take a More Direct Route [10]

Table 2.1 provides a summary of the constrained and unconstrained optimization classification based on direct/indirect search methods.

Table 2.1: Constrained and Unconstrained Optimization Classification

	Nonlinear - Unconstrained Optimization Techniques	Nonlinear - Constrained Optimization Techniques
Direct Search Methods	Random Search Methods - Random Jumping Method - Random Walk Method - Random Walk Method with Direction Exploitation Grid Search Method Univariate Method Pattern Directions Powell's Method Simplex Method	Random Search Methods Complex Method Sequential Linear Programming (Sequential Unconstrained Minimization Techniques – SUMT) Sequential Quadratic Programming Zoutendijk's Method of Feasible Directions Rosen's Gradient Projection Method Generalized Reduced Gradient Method
Indirect Search Method	Gradient of a Function Steepest Descent (Cauchy) Method Conjugate Gradient (Fletcher-Reeves) Method Newton's Method Marquardt Method Quasi-Newton Methods Davidon–Fletcher–Powell Method Broyden–Fletcher–Goldfarb–Shanno Method	Penalty Function Method - Interior Penalty Function Method - Exterior Penalty Function Method Extended Interior Penalty Function Methods Linear Extended Penalty Function Method Quadratic Extended Penalty Function Method Penalty Function Method for Parametric Constraints Augmented Lagrange Multiplier Method

2.3.2 Unconstrained Optimization

There are two popular iterative methods involved in unconstrained classification of optimization techniques: the conjugate gradient method (also referred to as the Fletcher-Reeves) and the Broyden- Fletcher-Goldfarb-Shanno (BFGS) method [5] [4].

The Fletcher-Reeves algorithm makes use of the conjugate search direction for finding the optimum solution of the objective function, using the information of the last iteration. The method produces the exact solution after a finite number of iterations and can be thought of as a direct method. The method efficiently minimizes a quadratic function in, at most, n steps (where

n is the size of the system matrix), while the computing process requires only small memory space.

The BFGS method is the most efficient variable metric method that could reach the desired search direction by gaining the information from the previous n iterations. The method makes an approximation to the inverse of the Hessian matrix, $H(x_0)^{-1}$ in equation, and uses specified updates of information by gradient evaluations. The BFGS method belongs to the classification of Quasi-Newton methods.

Unconstrained problems may arise when an appropriate penalty factor is implied in a constrained problem or the constraints are eliminated. The general form of an unconstrained problem is shown in Equation 2.8.

$$\text{Minimize } f(x_1, x_2, \dots, x_n) \quad (2.8)$$

Where $x = [x_1, x_2, \dots, x_n]^T$ is a column vector of n real-valued design variables.

2.3.3 Constrained Optimization

One of the most useful approaches for solving a constrained optimization problem is to convert a multi-dimensional constrained problem into a series of the equivalent one-dimensional unconstrained problem by using penalty and barrier methods.

Penalty methods are used to search for a feasible point that could satisfy all the constraints simultaneously. These techniques implement a penalty factor to the objective function, considering the effect of any constraint violations. Barrier methods are typically used to prevent the infeasible region from growing while we already have a feasible point relative to inequality constraints. In summary, the penalty method will drive the solution into a feasible region and the barrier method will keep the answer in the infeasible region (preventing from leaving the feasible region) [11].

2.3.3.1 Penalty Method

Considering the primary objective function as shown in Equation 2. 9:

$$\text{Minimize } f(x) \quad (2.9)$$

$$\begin{aligned} \text{Subject to } \quad & g_i(x) \leq 0, \quad i = 1, \dots, p \\ & h_j(x) = 0, \quad j = 1, \dots, m \end{aligned}$$

The penalized objective function can be formulated as Equation 2. 10.

$$\text{Minimize } f(x) \quad (2.10)$$

where

$$\begin{aligned} F(x, \rho, \beta) &= f(x) + P(x, \rho, \beta) \\ P(x) &= \sum_{j=1}^m \rho_j h_j^2(x) + \sum_{i=1}^p \beta_i g_i^2(x) \end{aligned}$$

Then, the penalty parameters ρ_j and β_i are given by Equation 2.11.

$$\begin{aligned} h_j &= \begin{cases} 0 & \text{if } h_j(x) = 0 \\ \rho_j \gg 0 & \text{if } h_j(x) \neq 0 \end{cases} \\ \beta_i &= \begin{cases} 0 & \text{if } g_i(x) \leq 0 \\ \beta_i \gg 0 & \text{if } g_i(x) > 0 \end{cases} \end{aligned} \quad (2.11)$$

2.3.3.2 Barrier Method

The goal of the barrier method is to prevent point x from ever approaching the boundary of the feasible region. The method introduces a barrier function $B(g(x)) \geq 0$, which is differentiable and approaches ∞ , as $g_i(x) \rightarrow 0^-$ and requires a start point that over satisfies the constraint.

For example:

$$-\sum_{j=1}^m \ln(-g_i(x)), b(x) = -\sum_{j=1}^m \frac{1}{g_i(x)}$$

Barrier function method is shown by Equation 2. 12.

$$B(\varepsilon^{(k)}) = \min \{f(x) + \varepsilon^{(k)} b(x)\} \Rightarrow x^{(k)} \quad (2.12)$$

$$\text{s.t. } g(x) < 0$$

Where the parameters sequence $\{\varepsilon^{(k)}\}_{k \in \mathbb{N}}$ satisfies the condition

$$0 < \varepsilon^{(k+1)} < \varepsilon^{(k)} \text{ and } \varepsilon^{(k)} \rightarrow 0.$$

2.3.3.3 Sequential Unconstrained Minimization Techniques (SUMT)

The Sequential Unconstrained Minimization Techniques (SUMT) method, presented by Fiacco and McCormick [12], is used to convert the constrained problem to the equivalent unconstrained optimization problem. SUMT implements the penalty method in order to find a suitable initial point, as well as taking advantage of the barrier method to make sure that the point will over satisfy the inequality constraints.

The steps of the SUMT search for the optimal solution are shown in the following sequence.

1. Choose the following parameters
 - R Initial value for r
 - θ Control rate of reduction r
 - ε Satisfactory error tolerance
 - k=Iteration number (set to be initially 0)
2. Search for an initial point x_0 that could over satisfy the inequality constraints and boundary conditions of an bjective function while being close enough to satisfy the equality constraints as well.
3. Form the barrier function $B(x, r)$, which consists of a specific term for each of the constraints and bounds. Combine the original form of an unconstrained objective function with this function to create the new objective equation. Minimize $F(x) = f(x) + B(x, r)$.
4. Find the optimal point x_{k+1} for the function $F(x)$ using the starting point x_k (using unconstrained Non-Linear Programming(NLP) methods, such as gradient base methods).
5. Consider the stopping conditions, *if* $|x_{k+1} - x_k| \leq \varepsilon$, then x_{k+1} is the optimal solution and the cycle is over. Otherwise, reduce the value of $r: r \leftarrow r\theta$, increase the value of $k: k \leftarrow k + 1$ and start from step 2 [12] [13].

2.3.3.4 Lagrange Multiplier

The Lagrange multiplier method can find the solution to optimization problems with equality constraints (Equation 2.13).

$$\text{Minimize } f(x) \quad (2.13)$$

$$\text{Subject to : } h_i(x) = 0, \quad j = 1, \dots, m < n$$

The Lagrange function can be formed as Equation 2.14.

$$L(x, \lambda) = f(x) + \sum_{j=1}^m \lambda_j h_j(x) = f(x) + \lambda^T h(x) \quad (2.14)$$

$$\frac{\partial L}{\partial x_i}(x^*, \lambda^*) = 0, \quad i = 1, \dots, n$$

$$\frac{\partial L}{\partial \lambda_j}(x^*, \lambda^*) = 0, \quad j = 1, \dots, m$$

Where λ represents $m \times 1$ vector of Lagrange multipliers, one for each constraint, x^* is the minimum solution, and λ^* is a set of associated Lagrange multipliers.

In case of a quadratic objective function and linear constraint, partial derivative could be found according to Equation 2.15.

$$L(x, \lambda) = \frac{1}{2} x^T A x + b^T x + c + \lambda^T (C x - d) \quad (2.15)$$

$$\frac{\partial L}{\partial x} = A x + b + C^T \lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = C x - d = 0$$

Results are presented in Equation 2.16 by the linear system, which can be solved directly.

$$\begin{bmatrix} A & C^T \\ C & 0 \end{bmatrix} \begin{bmatrix} x^* \\ \lambda^* \end{bmatrix} = \begin{bmatrix} -b^* \\ d \end{bmatrix} \quad (2.16)$$

2.3.4 Non-Gradient Based Methods

Gradient-free optimization methods are among the commonly applied numerical techniques used to find the local optimum of an objective function in a multidimensional space. Powell's conjugate direction method and the Nelder-Mead simplex algorithm are well-known approaches

in this classification. Both methods are capable of finding the local minimum in a non-linear, unconstrained optimization problem [14].

The Nelder-Mead simplex method performs a multi-dimensional search by heuristic algorithms. The method is efficient and the computational process is compact and robust, while it does not require the function to be derivate or smooth. The process simply makes use of a simplex (a triangle in ‘n’ dimensions) and generates a sequence of triangles in different shapes, for which the function values at the vertices get smaller and smaller [11].

2.4 Global Optimization Algorithms

“Global optimization is a branch of applied mathematics and numerical analysis that deals with the global optimization of a function or a set of functions according to some criteria” [15]. Typically, these techniques are proposed to return the best possible set of input values for which, the objective function generates the best optimal solution (among multiple optima), considering the constraints of the model.

In Figure 2.7, the minima at $x \approx -1$ represent the concept of local optimal (local minimum) in respect to the objective function $f(x)$, while $x=0$ represents a globally optimal solution (global minimum) for this function. Depending on the adopted method, all three points could be reached by local optimization methods.

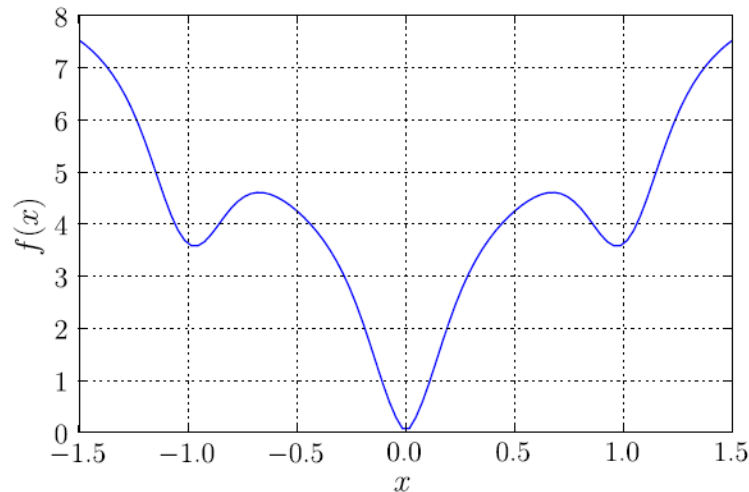


Figure 2.7: One Dimensional Multi-Model Function [9]

2.4.1 Classification and History

Thomas Weise [16] presented a rough taxonomy of global optimization methods in his book, named ‘Global Optimization Algorithms- Theory and Application’ presented, shown in Figure 2.8. Global optimization techniques are mainly classified as either deterministic or probabilistic algorithms.

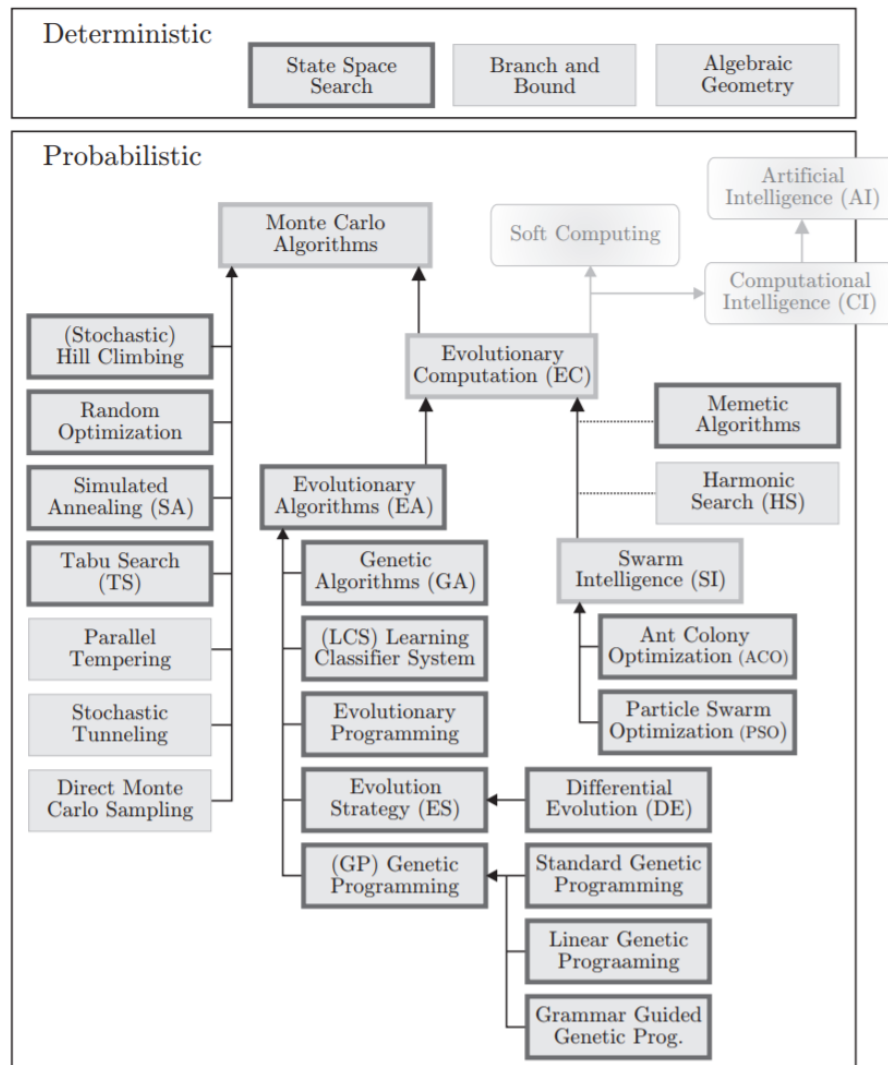


Figure 2.8: The Taxonomy of Global Optimization Algorithms [16]

A deterministic algorithm is a mathematical function that generally defines a unique set of solution to the specific set of input values in its domain. The deterministic method makes use of an explicit relation between the set of solution candidates and their fitness by exploring the search space. On the other hand, if the relation is very complicated or the search space has a large dimension, deterministic methods will no longer be efficient and probabilistic algorithms will be used to make the optimization feasible.

Probabilistic methods were refined 50 years ago; the most famous branch of this family are known to be Monte Carlo approaches. These techniques guarantee to improve the solutions in a short time, while the results might not be the global answer [16].

Heuristics algorithms are mathematical functions used in deterministic algorithms for global optimization to define the processing order of candidate solutions, while probabilistic methods may only rely on parameters that have been chosen by heuristic algorithms [16].

A metaheuristic method is a higher-level procedure, proposed to generate an efficient solution to an optimization problem. These algorithms, which are often nature-inspired, can be applied to solve complex optimization problems with uncertain, stochastic, dynamic information. For example, evolutionary algorithms are inspired by the behavior of biological evolution, and candidate solutions are considered as individuals competing in a virtual environment [17] [18]. Figure 2.9 presents different methods in each classification of metaheuristic groups which are the most popular metaheuristic methods between global optimization techniques.

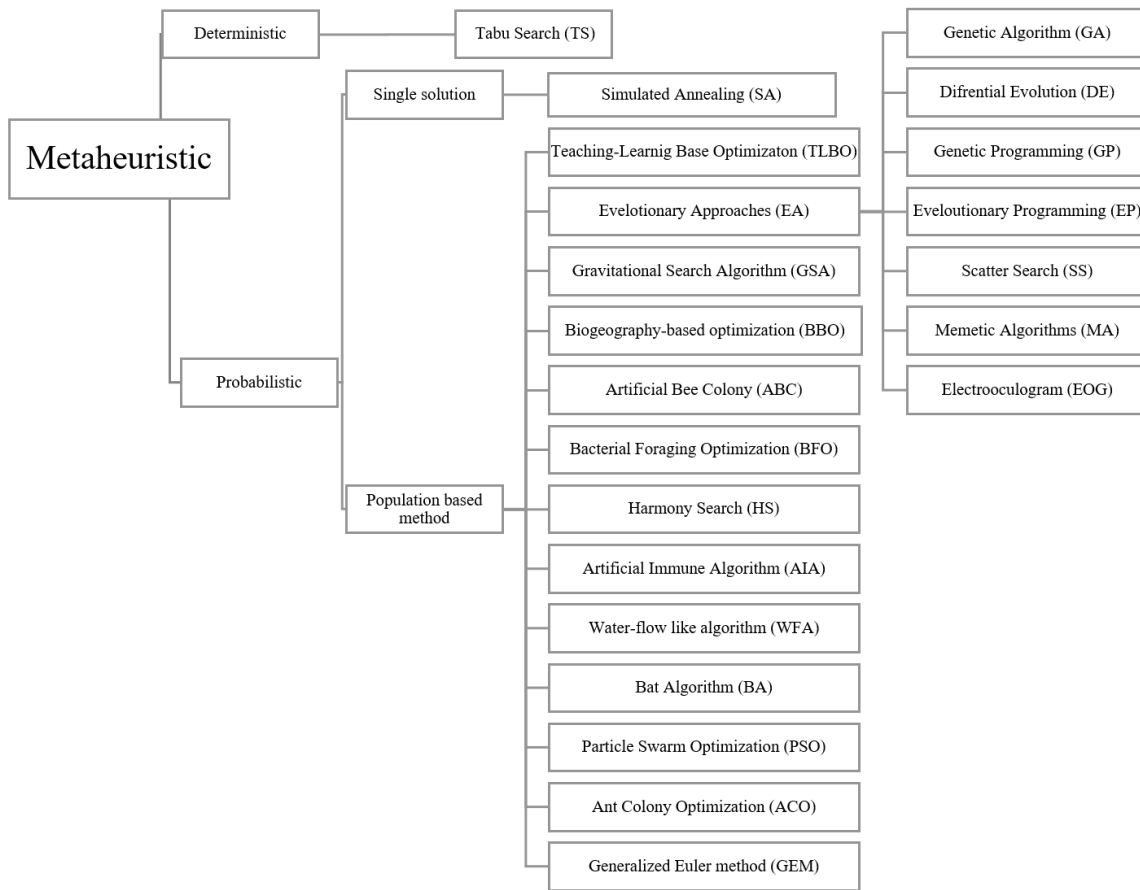


Figure 2.9: Metaheuristic Classification [19]

2.5 Teaching-Learning-Based Optimization (TLBO) algorithm

2.5.1 TLBO Overview

Teaching-Learning-Based Optimization (TLBO) algorithm is a popular method among metaheuristic techniques with relatively competitive performances.

The TLBO algorithm is a nature-inspired method proposed by Rao and colleagues [20]. This method is also a population-based algorithm which uses a population of solutions to proceed to the optimal solution. The population is considered as a group or class of learners. The most attracting point about the algorithm is that it does not need any algorithm-specific parameter to be tuned and only common controlling parameters like population size and the number of generations is required for its working. The method has gained wide acceptance for its simplicity and the ability to find the global or near-global optimum solutions in less number of function evaluations rather than other metaheuristic algorithms [21].

TLBO is inspired by the process of knowledge in the classroom environment. The method consists of two steps: ‘Teacher phase’ and ‘Learner Phase’. ‘Teacher Phase’ means acquiring knowledge from a teacher, and ‘Learner Phase’ means learning by interacting with classmates. The subjects offered to the learners are considered to be different design variables for the objective function of the optimization problem. The learners’ result is analogous to the ‘fitness’ of the optimization problem [20] [22]. The teacher is considered as the best optimal solution to the problem in the first step. TLBO focus on the influence of the teacher on the learners and the influence of learners on their classmates. The learners’ results are typically represented by the student’s mean grade. The teacher attempts to empower the students by increasing their knowledge and improving their grades. This will consequently result in a better class average based on his or her capabilities and teaching skills. On the other hand, despite the great efforts made by the teacher for increasing student’s grades, the quality of students sitting in the class cannot be ignored as an important factor that affects their grades. Quality of students is assessed through the mean value of the class population. Moreover, the teacher is responsible to make efforts to bring the students into a higher level of knowledge. A new teacher with better qualifications and skills would be required to teach the classroom at this higher level [20] [22].

Some notes have reported remarkable results about TLBO outperforming ES, PSO, ABC, DE, and GEM on a number of benchmarks and constrained optimization problems where TLBO reaches better or equal optimal solutions much faster than the other methods, as reported in [22] [23].

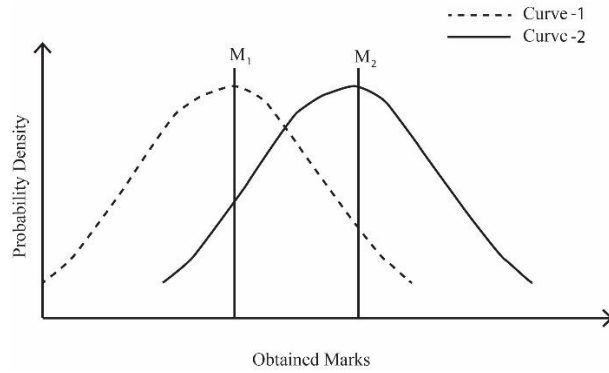


Figure 2.10: Model for the distribution of marks obtained by learners taught by two different teachers [20]

Figure 2.10 shows the distribution of marks achieved by the learners in two different classes [20]. Assuming that there are two different teachers for each class, T1 and T2, teaching the same subject to the same merit level learners in the classes. Curve 1 and Curve 2 represent the obtained marks, by learners taught by teacher T1 and T2 respectively, while both distributions are assumed to be normal. M1 and M2 represent the mean factors, as the main difference between both groups' results. As it can be seen in Figure 2.10, Curve 2 represents better marks obtained by students taught by teacher T2. Clearly, the comparison indicates that a better instructor conducts a better mean for the results of the learners.

The learners also improve their level of knowledge by interacting with other learners, which will increase their results.

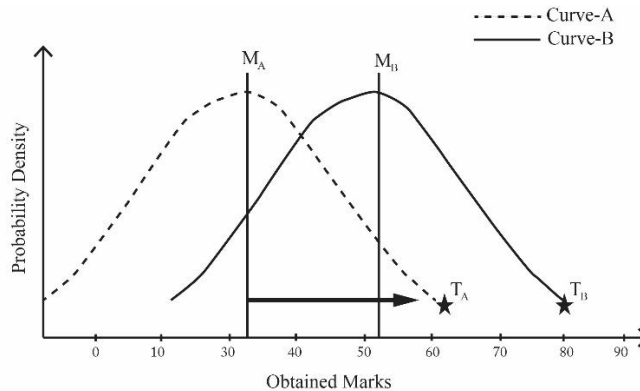


Figure 2.11: Model for the distribution of marks obtained for a group of learners [20]

Figure 2.11 illustrates a normal distribution model for the marks obtained by the learner for a class with Curve-A having mean MA. By considering the teacher as the most knowledgeable person in the group, the best learner in the society will be imitated as a teacher (TA in Figure 2.11). The teacher TA will try to improve the mean level of the class according to his or her teaching capabilities. Teacher TA will put maximum efforts to proceed the mean MA toward the new mean, MB, which is close to her or his level of knowledge depending on his or her capabilities; but the learners will gain the knowledge not only according to the quality of the teacher but also depending on the merit level of the students present in the class, which is judged from the mean value of the society. The quality of the students can improve to the stage MB by the efforts of teacher TA, where the learner requires a superior teacher with greater knowledge than themselves. Consequently, the algorithm will be followed by the new Curve-B with new teacher TB.

2.5.2 TLBO Algorithm

Table 2.2 presents the TLBO algorithm. As a population-based algorithm, the initial phase in TLBO starts with randomly generating a population of candidate solutions in the search space. The problem search space consists of n dimensions, and each dimension should satisfy its upper and lower boundaries. The process is divided into two parts namely: the ‘Teacher Phase’ and the ‘Learner Phase’ [20] [24].

- Teacher Phase

In this phase, the teacher tries to increase the knowledge of learners and increase the mean result of the class in the subject taught by him or her depending on his or her capability [20]. This procedure is exposed in the algorithm between lines (11–14) in Table 2.2. [22]

- Learner Phase

In this phase, the students improve their level of knowledge by interacting with other learners. A learner can improve his or her knowledge if the other randomly selected learner has more knowledge than him or her. The procedure for the learner phase is given in lines (27-28) in Table 2.2 [22].

Table 2.2: Algorithm (TLBO pseudocode) [22]

```

1       $n$ : dimension of the problem
2       $m$ : population size
3      MAXITER: maximum number of iterations
4      Initialization()
5      while ITER < MAXITER
6          Elite  $\leftarrow$  SelectBest(P,Elite)
7          for  $i = 1:m$ 
8               $T_F = \text{round}(1 + \text{rand})$ 
9               $X_{\text{mean}} \leftarrow \text{mean}(X_i)$ 
10              $X_{\text{teacher}} \leftarrow \text{best}(X_i)$ 
11              $X_{\text{new},i} = X_i + \text{rand} \cdot (X_{\text{teacher}} - (T_F \cdot X_{\text{mean}}))$ 
12             if  $f(X_{\text{new},i}) < f(X_i)$ 
13                  $X_i \leftarrow X_{\text{new},i}$ 
14             end if
15              $j \leftarrow \text{randi}(m)$ 
16             if  $j \neq i$ 
17                 if  $f(X_i) < f(X_j)$ 
18                      $X_{\text{new},i} = X_i + \text{rand} \cdot (X_i - X_j)$ 
19                 else
20                      $X_{\text{new},i} = X_i + \text{rand} \cdot (X_j - X_i)$ 
21                 end if
22             end if
23             if  $f(X_{\text{new},i}) < f(X_i)$ 
24                  $X_i \leftarrow X_{\text{new},i}$ 
25             end if
26         end for
27         P  $\leftarrow$  ReplaceWorstWithElite(P,Elite)
28         P  $\leftarrow$  RemoveDuplicateIndividuals(P)
29         ITER = ITER + 1
30     end while

```

Figure 2.12 presents the flowchart of Teaching-Learning-Based Optimization (TLBO) algorithm.

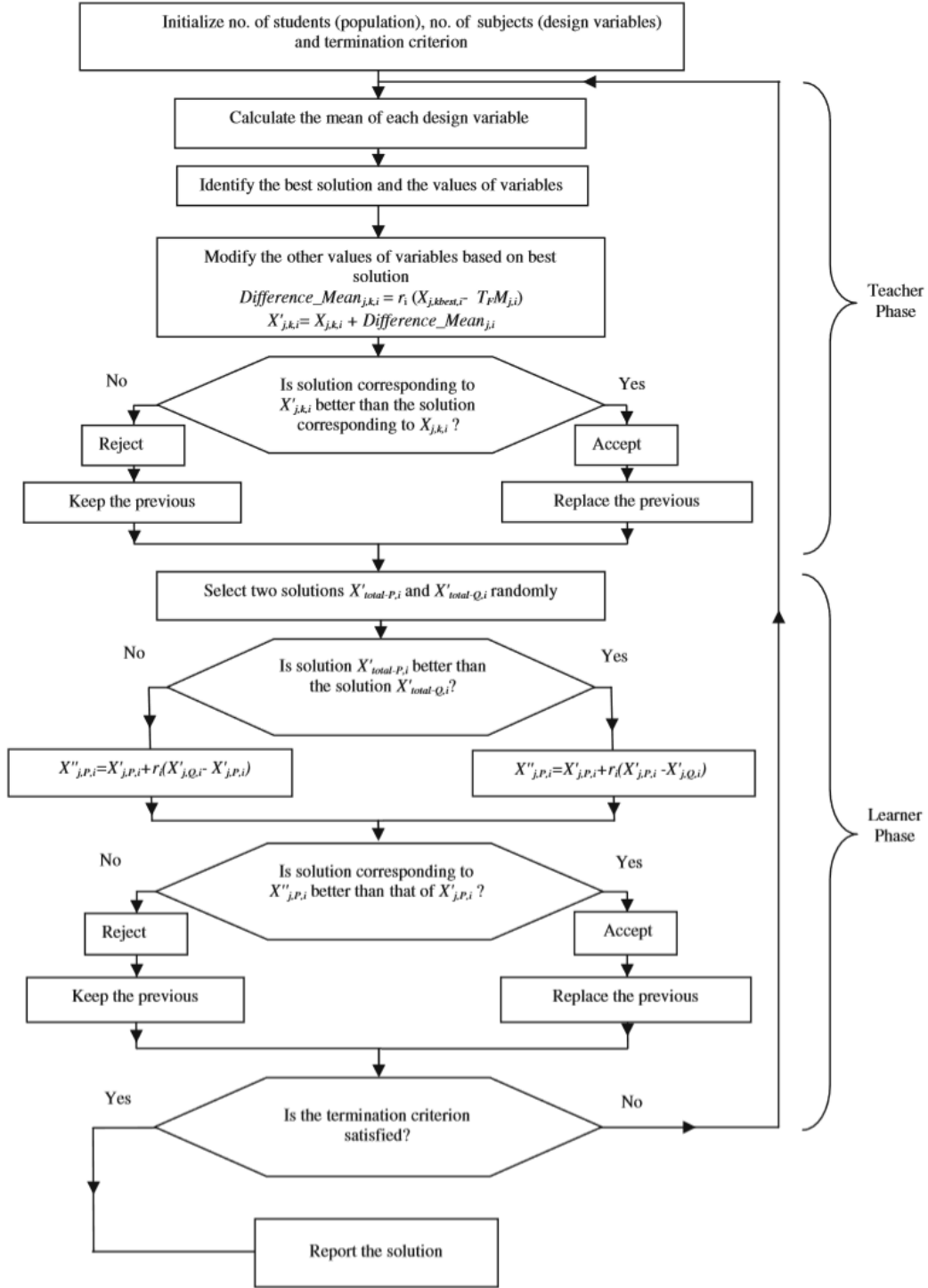


Figure 2.12: Flowchart of TLBO [20] [22]

2.5.3 Case Study of a Constrained Optimization Problem with TLBO Algorithm

In this case study, a constrained benchmark function of Himmelblau is solved with the TLBO algorithm adopted from [21]. The objective function needs to be minimized by values of x_1 and x_2 considering the given constraints.

$$\text{Minimize } f(x_i) = (x_1^2 + x_2 - 11)^2 + (x_1 + x_2^2 - 7)^2$$

Subject to:

$$g_1(x) = 26 - (x_1 - 5)^2 - x_2^2 \geq 0,$$

$$g_2(x) = 20 - 4x_1 - x_2 \geq 0,$$

$$-5 \leq x_1, x_2 \leq 5$$

The known solution to this benchmark function is 0 for $x_1 = 3$ and $x_2 = 2$ and $g_1(x) = 18$ and $g_2(x) = 16$ according to [21]. The population size is assumed to be five (numbers of learners). There are two design variables, x_1 and x_2 , and one iteration as the termination criterion.

Table 2.3 presents the initial population which is randomly generated within the ranges of variables and provides the corresponding values for the objective function. The mean values of x_1 and x_2 are calculated. The lowest value of $f(x)$ is considered as the best learner in this minimization problem (it is supposed as an equivalent to the teacher). The penalty factors will be assigned to the objective function in case of any violation to the given constraints upon to the designer/decision maker (in this case the penalty p_1 for violation of $g_1(x)$ is considered as $10 * (g_1(x))^2$ and the penalty p_2 for violation of $g_2(x)$ is considered as $10 * (g_2(x))^2$). The fitness function is called $f'(x)$ and is formulated according to Equation 2.17 as a minimization problem considering the penalty factors. The fitness function, $f'(x)$, is called the pseudo-objective function.

$$f'(x) = f(x) + 10 * (g_1(x))^2 + 10 * (g_2(x))^2 \quad (2.17)$$

Table 2.3: Initial population

x_1	x_2	$f(x)$	$g_1(x)$	p_1	$g_2(x)$	p_2	$f'(x)$
3.220	0.403	13.13922	22.66919	0	6.7170	0	13.13922
0.191	2.289	77.71054	-2.36600	55.9796	16.9470	0	133.69010
3.182	0.335	14.02423	22.58265	0	6.9370	0	14.02423
1.660	4.593	261.57319	-6.25124	390.7800	8.7670	0	652.35320
2.214	0.867	43.64116	17.48651	0	10.2770	0	43.64116
Mean=2.093 Mean=1.697							

In this step, the teacher tries to improve the mean result of the learners. The $difference_{mean}$ values for x_1 and x_2 are calculated by assuming the random numbers of $r_1 = 0.25$ and $r_2 = 0.43$ for x_1 and x_2 respectively and $T_f = 1$, shown by Equations (2.18) and (2.19):

$$difference_{mean}(x_1) = 0.25 * (3.22 - 2.093) = 0.2817 \quad (2.18)$$

$$difference_{mean}(x_2) = 0.43 * (0.403 - 1.697) = -0.55642 \quad (2.19)$$

The values under the x_1 and x_2 columns in Table 2.3 are added by the calculated $difference_{mean}$ values. Table 2.4 presents the new results for x_1 and x_2 , penalties and the values of the objective function and pseudo-objective function.

Table 2.4: New values of the variables, objective function and the penalties (teacher phase)

x_1	x_2	$f(x)$	$g_1(x)$	p_1	$g_2(x)$	p_2	$f'(x)$
3.50175	-0.15342	13.30313	23.7317	0	6.14642	0	13.30313
0.47275	1.73258	94.22118	2.50217	0	16.37642	0	94.22118
3.46375	-0.22142	12.76312	23.5909	0	6.36642	0	12.76312
1.94175	4.03658	136.437	0.35312	0	8.19642	0	136.437
2.49575	0.31058	39.32602	19.63227	0	9.70642	0	39.32602

Completing the teacher phase of the TLBO algorithm, the results of $f'(x)$ in Table 2.3 and 2.4 are compared and the best values are placed in Table 2.5.

Table 2.5: Updated values of the variables, objective function and the penalties based on fitness comparison (teacher phase)

x_1	x_2	$f(x)$	$g_1(x)$	p_1	$g_2(x)$	p_2	$f'(x)$
3.22	0.403	13.13922	22.66919	0	6.717	0	13.13922
0.47275	1.73258	94.22118	2.50217	0	16.37642	0	94.22118
3.46375	-0.22142	12.76312	23.5909	0	6.36642	0	12.76312
1.94175	4.03658	136.437	0.35312	0	8.19642	0	136.437
2.49575	0.31058	39.32602	19.63227	0	9.70642	0	39.32602

The learner phase considers the interaction between students to improve their knowledge. In this example, the interaction is assumed to have a random manner. Table 2.6 shows the new values of x_1 and x_2 in the first step of learner phase while the random numbers are considered to be $r_1 = 0.47$ for x_1 and $r_2 = 0.33$ for x_2 .

Table 2.6: New values of the variables, objective function and the penalties (learner phase)

x_1	x_2	$f(x)$	$g_1(x)$	p_1	$g_2(x)$	p_2	$f'(x)$	Interaction
4.5112	-0.03576	92.96006	25.75979	0	1.99096	0	92.96006	1 & 2
1.42356	1.26332	75.2906	11.61309	0	13.04244	0	75.2906	2 & 5
3.57831	-0.42747	12.38652	23.79606	0	6.11423	0	12.38625	3 & 1
2.20213	2.807	20.67471	10.29267	0	8.38448	0	20.67471	4 & 5
2.75613	-0.919	30.24144	20.12048	0	9.89448	0	30.24144	5 & 4

The results of $f'(x)$ in Tables 2.5 and 2.6 are compared and the best values are placed in Table 2.7 completing the learner phase.

Table 2.7: Updated values of the variables, objective function and the penalties based on fitness comparison

x_1	x_2	$f(x)$	$g_1(x)$	p_1	$g_2(x)$	p_2	$F'(x)$
3.22	0.403	13.13922	22.66919	0	6.7170	0	13.13922
1.42356	1.26322	75.2906	11.61309	0	13.04244	0	75.2906
3.57831	-0.42747	12.38652	23.79606	0	6.11423	0	12.38652
2.20213	2.807	20.67471	10.29267	0	8.38448	0	20.67471
2.75613	-0.919	30.24144	20.12048	0	9.89448	0	30.24144

As can be seen from the tables, the value of $f'(x)$ is minimized from 13.13922 at the beginning of iteration to 12.38652 at the end of the first iteration. The TLBO algorithm is able to reach the known results of this benchmark (0 for $x_1 = 3$ and $x_2 = 2$ and $g_1(x) = 18$ and $g_2(x) = 16$ according to [21]) by increasing the number of iterations. Computer programs such as

MATLAB, can be used to execute the TLBO algorithm for solving the unconstrained and constrained optimization problems.

2.6 More Case Studies of Optimization Problems

2.6.1 Case Study of Unconstrained Nonlinear Optimization Problem

The goal of this case study is to minimize the presented unconstrained function, by setting the partial derivatives equal to zero and solving the function by parameters value.

$$f(x_1, x_2, \dots, x_n) = x_1^2 + 2x_2^2 + 3x_3^2 + x_1x_2 + x_2x_3 - 8x_1 - 16x_2 - 32x_3 + 110$$

Equating the first derivatives to zero, result in

$$\frac{\partial f}{\partial x_1} = 2x_1 + x_2 - 8 = 0$$

$$\frac{\partial f}{\partial x_2} = x_1 + 4x_2 + x_3 - 16 = 0$$

$$\frac{\partial f}{\partial x_3} = x_2 + 6x_3 - 32 = 0$$

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 16 \\ 32 \end{bmatrix}$$

Using MATLAB and $X = A \backslash B$ command, the optimal solution for the function is resulted,

$$(\widehat{x}_1 \quad \widehat{x}_2 \quad \widehat{x}_3) = (3 \quad 2 \quad 5)$$

where the function value is:

$$f(3 \quad 2 \quad 5) = 2$$

To see if this point is the minimum, the Hessian matrix is formed.

$$H(\hat{x}) = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 6 \end{bmatrix}$$

Using MATLAB function eig(H), the eigenvalues are found to be 1.55, 4.0 and 6.45 which are all positive. Thus the Hessian matrix is a positive definite matrix and $(3 \quad 2 \quad 5)$ is a minimum point.

2.6.2 Case Study of Constrained Nonlinear Optimization Problem

2.6.2.1 Solution by Lagrange Multiplier Method

In this case study, the main propose is to apply Lagrange multiplier method for solving constrained parameter optimization to determine the minimum distance from the origin of the 'xy' plane to a circle described by:

$$\text{Constraint: } (x - 8)^2 + (y - 6)^2 = 25$$

The minimum distance is obtained by minimization of the distance square, given by:

$$\text{Objective Function: } f(x, y) = x^2 + y^2$$

The MATLAB plot command is used to plot the circle shown in Figure 2.13 [1].

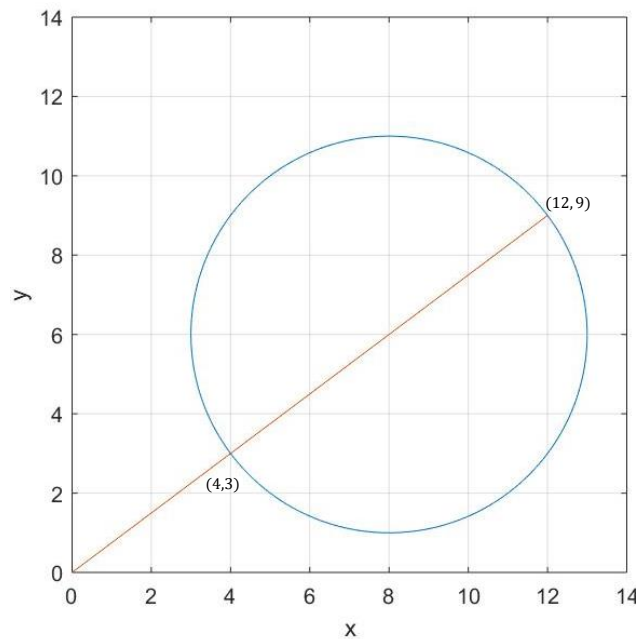


Figure 2.13: Constrained function of constrained nonlinear optimization problem case study

Forming the Lagrange function, we obtain:

$$\ell = x^2 + y^2 + \lambda[(x - 8)^2 + (y - 6)^2 - 25]$$

The necessary condition for extrema are:

$$\frac{\partial \ell}{\partial x} = 2x + \lambda(2x - 16) = 0$$

$$2x(\lambda + 1) = 16\lambda$$

$$\frac{\partial \ell}{\partial y} = 2y + \lambda(2y - 12) = 0$$

$$2y(\lambda + 1) = 12\lambda$$

$$\frac{\partial \ell}{\partial \lambda} = (x - 8)^2 + (y - 6)^2 - 25 = 0$$

Eliminating λ from the first two equations results in:

$$y = \frac{3}{4}x$$

Substituting in third equation yields:

$$\frac{25}{16}x^2 - 25x + 75 = 0$$

The solutions of above quadratic equations are $x = 12$ and $x = 4$. Thus, the corresponding extrema are at points (4,3) with $\lambda = 1$, and (12, 9) with $\lambda = -3$. It is clear that the minimum distance is at point (4,3) and the maximum is at point (12, 9). From this example, clearly, the minimum distance is 5, located at (4,3). To distinguish these points, the second derivatives are obtained and the Hessian matrix evaluated at these points are formed. The matrix with positive eigenvalues is a positive definite matrix and the parameters corresponds to the minimum point.

Table 2.8: Lagrange multiplier method, constrained optimization for the case study

	λ	X	Y	f (x)
Minimum Distance	1	4	3	25
Maximum Distance	-3	12	9	225

2.6.2.2 Solution by Newton-Raphson Method

In many optimization problems, the direct solution is not possible and the equation needs to be solved with iterative methods. A significantly superior method applicable for solving presented continues function in case study 2, is the Newton-Raphson method.

From the first two equations, x and y are found. These are

$$x = \frac{8\lambda}{\lambda + 1}$$
$$y = \frac{6\lambda}{\lambda + 1}$$

Substituting in third equation results in

$$f(\lambda) = \frac{100\lambda^2}{(\lambda + 1)^2} - \frac{200\lambda}{\lambda + 1} + 75 = 0$$

This is a non-linear equation in terms of λ and can be solved by the Newton-Raphson method.

$$\Delta\lambda^{(k)} = \frac{-\Delta f(\lambda)^{(k)}}{\left(\frac{df}{d\lambda}\right)^{(k)}}$$

$$\lambda^{(k+1)} = \lambda^{(k)} + \Delta\lambda^{(k)}$$

$$\frac{df(\lambda)}{d\lambda} = \frac{200\lambda}{(\lambda + 1)^3} - \frac{200}{(\lambda + 1)^2} = \frac{-200}{(\lambda + 1)^3}$$

The following commands show the procedure for the solution of the given case study, solved by the Newton-Raphson method and an initial estimate of $\lambda = 0.4$.

Table 2.9 and 2.10 summarize the results of this method. After five iterations the solution converges to $\lambda = 1.0, x = 4$, and $y = 3$, corresponding to the minimum length. If the program is run with the initial estimate of -2 , the solution converges to $\lambda = -3.0, x = 12$, and $y = 9$, which corresponds with maximum length [1].

Table 2.9: Newton-Raphson method and iterative solution, Lambda= 0.4

Enter estimated value of Lambda = 0.4

Iter	Df	J	DLambda	Lambda	x	y
1.0000	26.0204	-72.8863	0.3570	0.4000	2.2857	1.7143
2.0000	7.3934	-36.8735	0.2005	0.7570	3.4468	2.5851
3.0000	1.0972	-26.6637	0.0411	0.9575	3.9132	2.9349
4.0000	0.0337	-25.0505	0.0013	0.9987	3.9973	2.9980
5.0000	0.0000	-25.0001	0.0000	1.0000	4.0000	3.0000

Table 2.10: Newton-Raphson method and iterative solution, Lambda= -2

Enter estimated value of Lambda = -2

Iter	Df	J	DLambda	Lambda	x	y
1.0000	75.0000	200.0000	-0.3750	-2.0000	16.0000	12.0000
2.0000	27.8926	76.9346	-0.3625	-2.3750	13.8182	10.3636
3.0000	8.1227	38.1258	-0.2131	-2.7375	12.6042	9.4531
4.0000	1.2823	26.9480	-0.0476	-2.9506	12.1013	9.0760
5.0000	0.0454	25.0682	-0.0018	-2.9982	12.0036	9.0027
6.0000	0.0001	25.0001	-0.0000	-3.0000	12.0000	9.0000

The following commands show the MATLAB code for this case study, Newton-Raphson method [1].

```
% Iterative solution Using Newton method
iter = 0; % Iteration counter
Df = 10; % Error in Df is set to a high value
Lambda = input('Enter estimated value of Lambda = ');
fprintf('\n ')
disp([' Iter Df J DLambda Lambda' ...
' x y'])
while abs(Df) >= 0.0001 % Test for convergence
iter = iter + 1; % No. of iterations
x = 8*Lambda/(Lambda + 1);
```

```

y = 6*Lambda/(Lambda + 1);
Df = (x- 8)^2 + (y - 6)^2 - 25; % Residual
J = -200/(Lambda + 1)^3; % Derivative
Delambda =-Df/J; % Change in variable
disp([iter, Df, J, Delambda, Lambda, x, y])
Lambda = Lambda + Delambda; % Successive solution
End

```

2.6.3 MATLAB Optimization Toolbox

The Optimization Toolbox includes programs or m-files that can be used to solve different types of optimization problems. Many publications provide information on the optimization toolbox, including algorithms and examples for different programs [25].

The use of any program or m-file in the optimization toolbox requires the following steps:

- Selecting the appropriate program or m-file to solve the specific problem at hand.
- Formulation of the optimization problem in the format expected by MATLAB. In general, this involves stating the objective function in a specific form such as a “minimization” type and the constraints in a specific form such as “less than or equal to zero” type.
- The distinction between linear and nonlinear constraints.
- Identification of lower and upper bounds on design variables.
- Setting/changing the parameters of the optimization algorithm (based on the available options) [4], [25].

Each program or m-file in MATLAB can be implemented in several ways. For illustration, the help commands and the response for the program f-mincon are shown below. The function f-mincon can be used in 12 different ways as indicated below (by the help command). The differences depend on the available data in the mathematical model of the problem and the information required from the solution to the problem. In using the different function calls, any data missing in the mathematical model of the optimization problem needs to be indicated using a null vector as []. Note that the response is edited for brevity [4].

```
>> help fmincon
```

```

FMINCON finds a constrained minimum of a function of several
variables.

```

```

FMINCON attempts to solve problems of the form:

```

```

min F(X) subject to:

```

$A \cdot X \leq B, Aeq \cdot X = Beq$ (linear constraints)
 $C(X) \leq 0, Ceq(X) = 0$ (nonlinear constraints)
 $LB \leq X \leq UB$ (bounds)

$X = \text{FMINCON}(\text{FUN}, X0, A, B)$ $X = \text{FMINCON}(\text{FUN}, X0, A, B, Aeq, Beq)$ $X = \text{FMINCON}$
 $(\text{FUN}, X0, A, B, Aeq, Beq, LB, UB)$
 $X = \text{FMINCON}(\text{FUN}, X0, A, B, Aeq, Beq, LB, UB, \text{NONLCON})$
 $X = \text{FMINCON}(\text{FUN}, X0, A, B, Aeq, Beq, LB, UB, \text{NONLCON}, \text{OPTIONS})$
 $X = \text{FMINCON}(\text{FUN}, X0, A, B, Aeq, Beq, LB, UB, \text{NONLCON}, \text{OPTIONS}, \dots$
 $P1, P2, \dots)$ $[X, FVAL] = \text{FMINCON}(\text{FUN}, X0, \dots)$
 $[X, FVAL, \text{EXITFLAG}] = \text{FMINCON}(\text{FUN}, X0, \dots)$
 $[X, FVAL, \text{EXITFLAG}, \text{OUTPUT}] = \text{FMINCON}(\text{FUN}, X0, \dots)$
 $[X, FVAL, \text{EXITFLAG}, \text{OUTPUT}, \text{LAMBDA}] = \text{FMINCON}(\text{FUN}, X0, \dots)$
 $[X, FVAL, \text{EXITFLAG}, \text{OUTPUT}, \text{LAMBDA}, \text{GRAD}] = \text{FMINCON}(\text{FUN}, X0, \dots)$
 $[X, FVAL, \text{EXITFLAG}, \text{OUTPUT}, \text{LAMBDA}, \text{GRAD}, \text{HESSIAN}] = \text{FMINCON}$
 $(\text{FUN}, X0, \dots).$

Table 2.11 shows some of the MATLAB Optimization Toolbox commands [25].

Table 2.11: MATLAB Optimization Toolbox Commands [25]

MATLAB Programs or Functions for Solving Optimization Problems		
Type of optimization problem	Standard form for solution by MATLAB	Name of MATLAB program or function to solve the problem
Function of one variable or scalar minimization	Find x to minimize $f(x)$ with $x_1 < x < x_2$	fminbnd
Unconstrained minimization of function of several variables	Find \mathbf{x} to minimize $f(\mathbf{x})$	fminunc or fminsearch
Linear programming problem	Find \mathbf{x} to minimize $\mathbf{f}^T \mathbf{x}$ subject to $[\mathbf{A}]\mathbf{x} \leq \mathbf{b}$, $[\mathbf{A}_{eq}]\mathbf{x} = \mathbf{b}_{eq}$, $\mathbf{l} \leq \mathbf{x} \leq \mathbf{u}$	linprog
Quadratic programming problem	Find \mathbf{x} to minimize $\frac{1}{2}\mathbf{x}^T [\mathbf{H}]\mathbf{x} + \mathbf{f}^T \mathbf{x}$ subject to $[\mathbf{A}]\mathbf{x} \leq \mathbf{b}$, $[\mathbf{A}_{eq}]\mathbf{x} = \mathbf{b}_{eq}$, $\mathbf{l} \leq \mathbf{x} \leq \mathbf{u}$	quadprog
Minimization of function of several variables subject to constraints	Find \mathbf{x} to minimize $f(\mathbf{x})$ subject to $\mathbf{c}(\mathbf{x}) \leq \mathbf{0}$, $\mathbf{c}_{eq} = \mathbf{0}$ $[\mathbf{A}]\mathbf{x} \leq \mathbf{b}$, $[\mathbf{A}_{eq}]\mathbf{x} = \mathbf{b}_{eq}$, $\mathbf{l} \leq \mathbf{x} \leq \mathbf{u}$	fmincon
Goal attainment problem	Find \mathbf{x} and γ to minimize γ such that $F(\mathbf{x}) - \mathbf{w}\gamma \leq \mathbf{goal}$, $\mathbf{c}(\mathbf{x}) \leq \mathbf{0}$, $\mathbf{c}_{eq} = \mathbf{0}$ $[\mathbf{A}]\mathbf{x} \leq \mathbf{b}$, $[\mathbf{A}_{eq}]\mathbf{x} = \mathbf{b}_{eq}$, $\mathbf{l} \leq \mathbf{x} \leq \mathbf{u}$	fgoalattain
Minimax problem	Minimize $\max_{\mathbf{x}} [F_i(\mathbf{x})]$ such that $\mathbf{c}(\mathbf{x}) \leq \mathbf{0}$, $\mathbf{c}_{eq} = \mathbf{0}$ $[\mathbf{A}]\mathbf{x} \leq \mathbf{b}$, $[\mathbf{A}_{eq}]\mathbf{x} = \mathbf{b}_{eq}$, $\mathbf{l} \leq \mathbf{x} \leq \mathbf{u}$	fminimax
Binary integer programming problem	Find \mathbf{x} to minimize $\mathbf{f}^T \mathbf{x}$ subject to $[\mathbf{A}]\mathbf{x} \leq \mathbf{b}$, $[\mathbf{A}_{eq}]\mathbf{x} = \mathbf{b}_{eq}$, each component of \mathbf{x} is binary	bintprog

2.6.3.1 Case Study Using MATLAB Optimization Toolbox: Unconstrained Optimization with Objfun

Find the minimum of the Rosenbrock's parabolic valley function, given by:

$$f(x_1, x_2) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

starting from the initial point : $X_1 = \begin{Bmatrix} -1.2 \\ 1.0 \end{Bmatrix}$, $X^* = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$

$$f_1 = 24.0 \quad f^* = 0.0$$

Solution

Step 1: Writing an M-file objfun.m for the objective function.

```
function f= objfun (x)
f= 100* (x(2)-x(1) *x(1)) ^2+(1-x(1)) ^2;
```

Step 2: Invoking the unconstrained optimization program in a new MATLAB file.

```
clc
clear all
warning off
x0 = [-1.2,1.0]; % Starting guess
f=objfun(x0)
options = optimset('LargeScale', 'off');
[x, fval] = fminunc (@objfun,x0,options)
```

The values of function value at the starting point is $f = 24.2000$. Optimization completed because the size of the gradient is less than the default value of the optimality tolerance. The final result is obtained for $x = [1.0000 \quad 1.0000]$ where the minimum of the Rosenbrock's parabolic valley function is $2.8358e-11$.

2.6.3.2 Case Study of Multivariable Minimization Problems Using MATLAB Function F-mincon

The solution of multivariable minimization problems, with inequality and equality constraints, using the MATLAB function f-mincon is illustrated in this section [4].

Find the solution of presented function starting from the initial point $X_1 = \{0.1 \ 0.1 \ 3.0\}^T$

$$\text{Minimize } f(x) = x_1^3 - 6x_1^2 + 11x_1 + x_3$$

Subject to

$$\begin{aligned}x_1^2 + x_2^2 - x_3^2 &\leq 0 \\4 - x_1^2 - x_2^2 - x_3^2 &\leq 0 \\x_3 - 5 &\leq 0 \\-x_i &\leq 0, i = 1, 2, 3\end{aligned}$$

Solution

Step 1: Write an M-file objfun.m for the objective function.

```
function f= objfun (x)
f= x(1)^3-6*x(1)^2+11*x(1)+x(3);
```

Step 2: Write an M-file constraints.m for the constraints.

```
function [c, ceq] = constraints (x)
% Nonlinear inequality constraints
c = [x(1)^2+x(2)^2-x(3)^2; 4-x(1)^2-x(2)^2-x(3)^2; x(3)-5; -
x(1); -x(2); -x(3)];
% Nonlinear equality constraints
ceq = [];
```

Step 3: Invoke constrained optimization program (write this in new MATLAB file).

```
clc
clear all
warning off
x0 = [.1, .1, 3.0];
% Starting guess
f=objfun (x0)
[c, ceq] = constraints (x0)
options = optimset ('LargeScale', 'off');
```

```
[x, fval]=fmincon (@objfun, x0, [], [], [], [], [], [], [],
@constraints, options)
[c, ceq] = constraints (x) % Check the constraint values at x
```

The results for function value and constraints at the starting point is presented in Table 2.12. Local minimum found that satisfies the constraints. According to solver stopping criteria details, the optimization completed because the objective function is non-decreasing in feasible directions. Constraints are satisfied within the default value of the constraint tolerance.

Table 2.12 summarizes the results achieved with MATLAB optimization toolbox using the fmincon function.

Table 2.12: Results for Multivariable Minimization Problem Using MATLAB Function fmincon

		Initial Point	Optimum Solution
Variables	x_1	0.1	0
	x_2	0.1	1.4142
	x_3	3	1.4142
Constraints	$x_1^2 + x_2^2 - x_3^2 \leq 0$	-8.98	0
	$4 - x_1^2 - x_2^2 - x_3^2 \leq 0$	-5.02	0
	$x_3 - 5 \leq 0$	-2	-3.5858
	$-x_1 \leq 0$	-0.1	0
	$-x_2 \leq 0$	-0.1	-1.4142
	$-x_3 \leq 0$	-3	-1.4142
	$f(x)$		
	<i>Minimize</i> $f(x) = x_1^3 - 6x_1^2 + 11x_1 + x_3$	4.041	1.4142

2.7 Conclusions

This chapter has provided an overview of optimization techniques typically used in engineering optimization applications. Optimization methods are being applied in a wide spectrum of industries and have reached a degree of maturity with rapid advances in computational techniques. In recent years, large-scale engineering systems, with multi-objective functions and a set of constraints, can be optimized with both accuracy and efficiency.

In this chapter, the mathematical principles and foundations behind the optimization algorithms were discussed. The methods were classified as either local or global algorithm, while both constrained and unconstrained optimization problems were considered. The knowledge behind available algorithms would help the decision-maker choose the appropriate method, based on the optimization case at hand. Three numerical case studies with constraint and unconstrained objective function have been discussed. In the next chapter, the application of these studies in power systems will be explained explicitly.

Chapter 3

3. Optimal Power Flow

3.0 Introduction

The Optimal Power Flow (OPF) problem has a long research history spanning more than 45 years. A comprehensive model and formulation of the economic dispatch problem including voltage and operating constraints were introduced and it was later named the optimal power flow problem.

The main focus of OPF studies is based on the minimization of operational cost while considering the system constraints. There have been some efforts to point out other objective functions of the OPF problem, such as minimization of active and reactive power losses, variation in deregulated electricity markets, voltage stability indexes and minimization of emission index in recent years [26].

The main objective of this chapter is to provide a brief review of the Economic Dispatch (ED) problem and introduce the lambda iteration method through a few case studies. The load flow problem is presented and a comprehensive model of the OPF problem with different states has been represented. The concept of Security Constraint Optimal Power Flow (SCOPF) in a power system has been discussed and simulated on two standard case studies.

3.1 Power Flow

Power flow studies are the backbone of power system analysis and design. They are necessary for planning and designing the future expansion of power systems as well as determining the best operation of existing systems. In addition, power flow analysis is required for many analyses, such as transient stability and contingency studies. The principal information obtained from a power flow study is the magnitude and phase angle of the voltage at each bus and the real and reactive power flow in each line. In any interconnected power system of N buses,

the power injections at the buses are given by a set of $2N$ nonlinear simultaneous equations presented in order by Equations 3.1 and 3.2 [1] [27].

$$P_G^k - P_D^k = V_k \sum_{i=1}^N \left[V_i [G_{ij}^k \cos(\delta^k - \delta^i) + B_{ij}^k \sin(\delta^k - \delta^i)] \right] \quad (3.1)$$

$$Q_G^k - Q_D^k = V_k \sum_{i=1}^N \left[V_i [G_{ij}^k \sin(\delta^k - \delta^i) - B_{ij}^k \cos(\delta^k - \delta^i)] \right] \quad (3.2)$$

Where $k = 1, 2, \dots, N$

V_k is the voltage magnitude at bus k

δ^k is the voltage angle at bus k

P_G^k is the active power generation at bus k

P_D^k is the active power demand at bus k

Q_G^k is the reactive power generation at bus k

Q_D^k is reactive power demand at bus k

G_{ij}^k is the real part of (k, j) element of the bus admittance matrix

B_{ij}^k is the imaginary part of (k, j) element of the bus admittance matrix

Generally, the goal of power flow studies is to determine the specification of three different types of buses in the network. At each bus, two of four quantities δ_i , $|V_i|$, P_i , and Q_i are specified, and the remaining two are calculated.

3.1.1 Load Buses

A load bus or PQ bus is a non-generator bus where the total amount of active and reactive power generation is zero, usually the real power and reactive power of the load are known (according to previous records, load forecast or measurements). The unknown quantities are δ_i , $|V_i|$, which need to be calculated. This is usually a generator bus.

3.1.2 Voltage Controlled Buses

At voltage controlled buses, the voltage magnitude is kept at a constant and is generally called the generator bus or PV bus. Both of the quantities, $|V_i|$ and P_{G_i} , are specified and the δ_i and Q_i are the two unknown quantities that must be determined by power flow analysis.

3.1.3 Slack Bus

At the slack bus, the voltage angle δ_k is set to zero as a reference for the angles of all other bus voltages in the network. Figure 3.1 shows a bus schematic. The voltage magnitude of the slack bus $|V_k|$ is also specified. This bus has the ability to either absorb or supply active or reactive power. The other two unknown quantities, P_G^k and Q_G^k , should be calculated by power flow analysis [1].

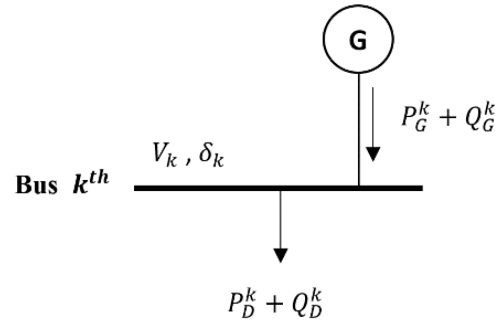


Figure 3.1: A typical bus in a network [1]

3.2 Economic Dispatch (ED)

The main objective of the Economic Dispatch (ED) problem is to minimize the total fuel cost of generation. The problem is subject to meet a set of equality constraints with respect to load and generation balances. The upper and lower limits on the output of generating units would provide a set of inequality constraints which needs to be satisfied by the Economic Dispatch solution.

3.2.1 Economic Dispatch Neglecting Losses and No Generator Limits

The simplest scenario for solving the Economic Dispatch problem neglects transmission losses. In this case, the problem does not consider system configuration and line impedance [1] [27]. This model assumes that the system consists of only one bus where all the generators and loads are connected. Figure 3.2 shows the shared bus in the simplest case of Economic Dispatch. The Economic Dispatch is the simplest model used when the transmission distance is very short and the load density is very high.

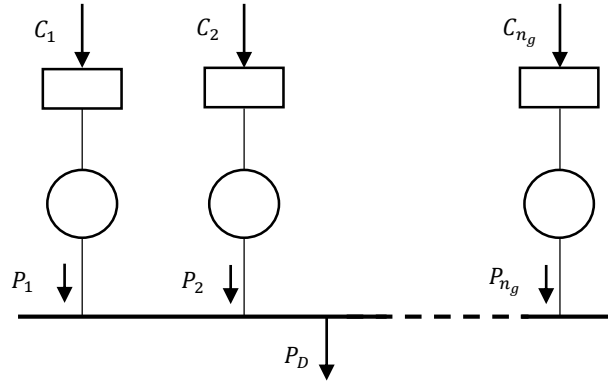


Figure 3.2: Economic Dispatch, Considering All Generators in One Shared Bus [1]

Since transmission lines are neglected, the total demand, P_D , is the sum of all generation. A cost function, C_i , is assumed to be known for each plant such that the objective function (total production cost) is minimum and subject to the constraint as defined by the Equations 3.3 and 3.4 [1].

$$\text{Minimize } C_t = \sum_{i=1}^{n_g} C_i = \sum_{i=1}^n (\alpha_i + \beta_i P_i + \gamma_i P_i^2) \quad (3.3)$$

$$\text{Subject to: } \sum_{i=1}^{n_g} P_i = P_D \quad (3.4)$$

C_t = total fuel cost

P_D = total demand

P_i = generation of i th plant

n_g = total number of dispatchable generating plants.

A typical approach is to augment the constraints into an objective function by using the Lagrange multipliers according to Equation 3.5.

$$\mathcal{L} = C_t + \lambda (P_D - \sum_{i=1}^{n_g} P_i) \quad (3.5)$$

The minimum of this unconstrained function is found where the derivative of the function to its variables is zero (Equation 3.6).

$$\frac{\partial \mathcal{L}}{\partial P_i} = 0 \Rightarrow \frac{\partial C_t}{\partial P_i} + \lambda(0 - 1) = 0 \xrightarrow{C_t = C_1 + C_2 + \dots + C_{n_g}} \frac{\partial C_t}{\partial P_i} = \frac{dC_i}{dP_i} = \lambda \quad (3.6)$$

Therefore all units have the same incremental operating cost, denoted here by λ , in order to minimize the total operating cost, C_t .

The coordination equation can be expressed as Equation 3.7.

$$P_i = \frac{\lambda - \beta_i}{2\gamma_i} \quad (3.7)$$

A simple example of the economic dispatch problem neglecting losses and generation limits is presented below. The example is cited from the Saadat textbook [1].

The fuel cost functions for three thermal plants are given by:

$$\begin{aligned} C_1 &= 500 + 5.3 P_1 + 0.004 P_1^2 & \$/hr \\ C_2 &= 400 + 5.5 P_2 + 0.006 P_2^2 & \$/hr \\ C_3 &= 200 + 5.8 P_3 + 0.009 P_3^2 & \$/hr \end{aligned}$$

Where P_1 , P_2 , and P_3 are in Megawatts (MW). The total load demand is 800 MW. Neglecting line losses and generator limits, the economic dispatch results and the total cost in \$/hr can be found by using either analytical or iterative methods [1]. The analytical solution uses the derivative of cost functions using the Equation 3.6, in order to find the same incremental operating cost according to the following equation:

$$\frac{\partial C_1}{\partial P_1} = 5.3 + 0.008 P_1 = \lambda$$

$$\frac{\partial C_2}{\partial P_2} = 5.5 + 0.012 P_2 = \lambda$$

$$\frac{\partial C_3}{\partial P_3} = 5.8 + 0.018 P_3 = \lambda$$

$$\lambda = \frac{\partial C_1}{\partial P_1} = \frac{\partial C_2}{\partial P_2} = \frac{\partial C_3}{\partial P_3}$$

$$P_1 + P_2 + P_3 = 800 \text{ MW}$$

Converting the above equations to matrix form:

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0.008 & 0 & 0 & -1 \\ 0 & 0.012 & 0 & -1 \\ 0 & 0 & 0.018 & -1 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ \lambda \end{bmatrix} = \begin{bmatrix} 800 \\ -5.3 \\ -5.5 \\ -5.8 \end{bmatrix}$$

Then the dispatch generation of each unit and the incremental fuel cost are:

$$P_1 = 400 \text{ MW}$$

$$P_2 = 250 \text{ MW}$$

$$P_3 = 150 \text{ MW}$$

$$\lambda = 8.5 \text{ \$/MWh}$$

The total fuel cost is:

$$C_T = C_1 + C_2 + C_3$$

$$C_T = 500 + 5.3 (400) + 0.004 (400)^2 + 400 + 5.5 (250) + 0.006 (250)^2 + 200$$

$$+ 5.8 (150) + 0.009 (150)^2 = 6,682.5 \text{ $/h}$$

The iterative solution relies on the gradient method and uses an initial value for the coordination equation.

In this case, the initial value is assumed to be $\lambda^{(1)} = 6.0 \text{ $/MWh}$. To form the coordination equations using Equation 3.5, we will have:

$$P_1^{(1)} = \frac{\lambda - \beta_1}{2\gamma_1} = \frac{6.0 - 5.3}{2 \times 0.004} = 87.5000 \text{ MW}$$

$$P_2^{(1)} = \frac{\lambda - \beta_2}{2\gamma_2} = \frac{6.0 - 5.5}{2 \times 0.006} = 41.6667 \text{ MW}$$

$$P_3^{(1)} = \frac{\lambda - \beta_3}{2\gamma_3} = \frac{6.0 - 5.8}{2 \times 0.009} = 11.1111 \text{ MW}$$

Since $P_D = 800 \text{ MW}$ the error ΔP is:

$$\Delta P^{(1)} = 800 - (87.5 + 41.6667 + 11.1111) = 659.7222 \text{ MW}$$

$$\Delta \lambda^{(1)} = \frac{659.7222}{\frac{1}{2 \times 0.004} + \frac{1}{2 \times 0.006} + \frac{1}{2 \times 0.009}} = \frac{659.7222}{263.8888} = 2.5 \text{ $/MWh}$$

$$\lambda^{(2)} = 6.0 + 2.5 = 8.5 \text{ $/MWh}$$

$$P_1^{(2)} = \frac{\lambda - \beta_1}{2\gamma_1} = \frac{8.5 - 5.3}{2 \times 0.004} = 400 \text{ MW}$$

$$P_2^{(2)} = \frac{\lambda - \beta_2}{2\gamma_2} = \frac{8.5 - 5.5}{2 \times 0.006} = 250 \text{ MW}$$

$$P_3^{(2)} = \frac{\lambda - \beta_3}{2\gamma_3} = \frac{8.5 - 5.8}{2 \times 0.009} = 150 \text{ MW}$$

$$\Delta P^{(2)} = 800 - (400 + 250 + 150) = 0 \text{ MW}$$

Since $\Delta P^{(2)} = 0$, the equality constraint is met in two iteration. Therefore, the economic dispatch results are:

$$P_1 = 400 \text{ MW}$$

$$P_2 = 250 \text{ MW}$$

$$P_3 = 150 \text{ MW}$$

The total fuel cost is:

$$\begin{aligned} C_t &= C_1 + C_2 + C_3 \\ &= 500 + 5.3 (400) + 0.004 (400)^2 + 400 + 5.5 (250) + 0.006 (250)^2 \\ &\quad + 200 + 5.8 (150) + 0.009 (150)^2 = 6,682.5 \quad \$/hr \end{aligned}$$

3.2.2 Economic Dispatch Neglecting Losses and Including Generator Limits

Considering the generation limits of a power plant, the power output of any generator should not exceed its rating, nor should it be below the necessary rating for stable boiler operations. Thus, the generations are restricted to lie within given minimum and maximum limits. The problem is finding the real power generation for each generator such that the total fuel cost function is minimum, that it is subject to the equality constraint given by Equation 3.4 and the inequality constraints given by Equation 3.8 [1] [27].

$$P_{i(min)} \leq P_i \leq P_{i(max)} \quad i = 1, \dots, n_g \quad (3.8)$$

Where $P_{i(min)}$ is the minimum generating limit for plant i and $P_{i(max)}$ is the maximum generating limit for plant i .

The numerical solution for an estimated λ , can be found from the coordination equation which is presented by Equation 3.7, and iteration is continued until Equation 3.4 is satisfied. As soon as any plant reaches its maximum or minimum power generation limits, the plant becomes kept at the limit. In effect, the plant output becomes a constant, and only the unviolated plants must operate at an equal incremental cost [1].

A simple example of the economic dispatch problem neglecting losses and including generation limits is presented below. This example is cited from Glover's textbook [27].

An area of an interconnected power system has two fossil-fuel units operating on economic dispatch. The variable operating costs of these units are given by:

$$\begin{aligned} C_1 &= 10P_1 + 8 \times 10^{-3}P_1^2 \quad \$/hr \quad 100 \leq P_{G1} \leq 600 \text{ MW} \\ C_2 &= 8P_2 + 9 \times 10^{-3}P_2^2 \quad \$/hr \quad 400 \leq P_{G1} \leq 1000 \text{ MW} \end{aligned}$$

Where P_1 and P_2 are in Mega Watts (MW).

The goal is to determine the power output and the incremental operating cost of each unit such that the results could minimize the total operating cost function (C_T), while the total load demand (P_D) varies between 500MW to 1500 MW. The generating unit inequality constraints will be considered and transmission losses can be neglected [27].

At light loads, unit 2 operates at its lower limit of 400 MW, where its incremental operating cost is $\frac{dC_2}{dP_2} = 15.2$ \$/MWh. In this case, the additional load comes from unit1 until $\frac{dC_1}{dP_1} = 15.2$ \$/MWh.

$$\begin{aligned} \frac{dC_1}{dP_1} &= 10 + 16 \times 10^{-3}P_1 = 15.2 \text{ $/MWh} \\ P_1 &= 325 \text{ MW} \end{aligned}$$

At this point the total power is : $P_T = 325 + 400 = 725 \text{ MW}$

For P_T less than 725 MW, where P_1 is less than 325 MW, the incremental operating fuel cost of the area is determined by unit 1 alone.

At heavy loads, unit 1 operates at its upper limit of 600 MW, where its incremental operating cost is $\frac{dC_1}{dP_1} = 19.6$ \$/MWh. Additional load comes from unit 2 for all values of $\frac{dC_2}{dP_2}$ greater than 19.6 \$/MWh. At $\frac{dC_2}{dP_2} = 19.6$ \$/MWh :

$$\begin{aligned} \frac{dC_2}{dP_2} &= 8 + 18 \times 10^{-3}P_2 = 19.60 \text{ $/MWh} \\ P_2 &= 644 \text{ MW} \end{aligned}$$

At this point the total power is : $P_T = 600 + 644 = 1244 \text{ MW}$

For P_T greater than 1244 MW, where P_2 is greater than 644 MW, the incremental operating fuel cost of the area is determined only by unit 2 .

For $725 \text{ MW} < P_T < 1244 \text{ MW}$, neither of the units has reached a limit value and the economic dispatch solution is the same as values given in Table 3.1 [27].

Table 3.1 summarizes the results of the economic dispatch problem which considers the generator limits while neglecting the transmission losses in this example [27].

Table 3.1: Economic Dispatch Solution Neglecting Losses and Including Generator Limits [27]

P_L	P_{G1}	P_{G2}	$\lambda = \frac{dC}{dP}$	C_T
MW	MW	MW	\$/MWhr	\$/hr
500	100	400	11.60	5720
600	200	400	13.20	6960
700	300	400	14.80	8360
725	325	400	15.20	8735
800	365	435	15.84	9899
900	418	482	16.68	11,525
1000	471	529	17.53	13,235
1100	524	576	18.38	15,030
1200	576	624	19.22	16,910
1244	600	644	19.60	17,765
1300	600	700	20.60	18,890
1400	600	800	22.40	21,040
1500	600	900	24.20	23,370

3.2.3 Economic Dispatch Including Generator Limits and Losses

When transmission distances are very small and load density is very high, transmission losses may be neglected and the optimal dispatch of generation is achieved with all plants operating at the equal incremental production cost. However, in a large interconnected network where power is transmitted over long distances with low load density areas, transmission losses are a major factor and they affect the optimum dispatch of generation. One common practice for including the transmission line losses is to express the total transmission loss as a quadratic function of the generator power outputs [1] [27].

The basic quadratic form is the same as Equation 3.3 while considering 3.8 and 3.9:

$$C_t = \sum_{i=1}^{n_g} C_i = \sum_{i=1}^n \alpha_i + \beta_i P_i + \gamma_i P_i^2 \quad (3.3)$$

$$P_{i(min)} \leq P_i \leq P_{i(max)} \quad i = 1, \dots, n_g \quad (3.8)$$

$$\sum_{i=1}^{n_g} P_i = P_D + P_L \quad (3.9)$$

Where $P_{i(min)}$ and $P_{i(max)}$ are the minimum and maximum generating limits for plant i respectively. The common approach is to augment the constraints into the objective function by using the Lagrange multipliers which is shown by Equation 3.10:

$$\mathcal{L} = C_t + \lambda (P_D + P_L - \sum_{i=1}^{n_g} P_i) + \sum_{i=1}^{n_g} \mu_{i(max)} (P_i - P_{i(max)}) + \sum_{i=1}^{n_g} \mu_{i(min)} (P_i - P_{i(min)}) \quad (3.10)$$

The minimum of this unconstrained function is found where the derivative of the function to its variables are zero according to Equation 3.11 [1].

$$\left\{ \begin{array}{l} \frac{\partial \mathcal{L}}{\partial P_i} = 0 \\ \frac{\partial \mathcal{L}}{\partial \lambda} = 0 \\ \frac{\partial \mathcal{L}}{\partial \mu_{i(max)}} = P_i - P_{i(max)} = 0 \\ \frac{\partial \mathcal{L}}{\partial \mu_{i(min)}} = P_i - P_{i(min)} = 0 \\ \frac{\partial C_t}{\partial P_i} + \lambda \left(0 - \frac{\partial P_L}{\partial P_i} - 1 \right) = 0 \end{array} \right. \quad (3.11)$$

Also,

$$C_t = C_1 + C_2 + \dots + C_{n_g} \quad (3.12)$$

$$\frac{\partial C_t}{\partial P_i} = \frac{\partial C_i}{\partial P_i} \quad (3.13)$$

$$\frac{\partial C_i}{\partial P_i} + \lambda \frac{\partial P_L}{\partial P_i} = \lambda \quad i = 1, \dots, n_g \quad (3.14)$$

The term $\frac{\partial P_L}{\partial P_i}$ in Equation 3.14 is known as the incremental transmission loss. From the substitution of equation $\frac{\partial \mathcal{L}}{\partial \lambda} = 0$ in Equation 3.10, as well as consideration of Equation 3.9, Equation 3.14 will be derived. This equation can be simplified in the form of Equation 3.15 [1].

$$\frac{\partial C_t}{\partial P_i} \left(\frac{1}{1 - \frac{\partial P_L}{\partial P_i}} \right) = \lambda \quad i = 1, \dots, n_g \quad (3.15)$$

or $L_i \frac{\partial C_t}{\partial P_i} = \lambda \quad i = 1, \dots, n_g$

Where L_i , is known as a penalty factor of plant i and is given by Equation 3.16:

$$L_i = \frac{1}{1 - \frac{\partial P_L}{\partial P_i}} \quad (3.16)$$

An example of the economic dispatch problem that considers generation limits and transmission losses is presented below. In this example, the generators' cost functions, the transmission loss model and the incremental fuel cost for the area are provided by the given information.

An area of an interconnected power system has two fossil-fuel units operating on economic dispatch. The variable operating costs of these units are given by [27]:

$$C_1 = 10P_1 + 8 \times 10^{-3}P_1^2 \quad \$/hr \quad 100 \leq P_{G1} \leq 600 \text{ MW}$$

$$C_2 = 8P_2 + 9 \times 10^{-3}P_2^2 \quad \$/hr \quad 400 \leq P_{G1} \leq 1000 \text{ MW}$$

and the transmission loss model is given by:

$$P_L = 1.5 \times 10^{-4}P_1^2 + 2 \times 10^{-5}P_1P_2 + 3 \times 10^{-5}P_2^2$$

Where P_1 and P_2 are in megawatts. The goal of this example is to determine the power output of each unit, the total transmission losses, the total load demand, and the total operating cost, C_T , when the area $\lambda = 16.00 \text{ \$/MWh}$. Substituting the following model in Equation 3.15:

$$\frac{\partial C_1}{\partial P_1} \left(\frac{1}{1 - \frac{\partial P_L}{\partial P_1}} \right) = \frac{10 + 16 \times 10^{-3}P_1}{1 - (3 \times 10^{-4}P_1 + 2 \times 10^{-5}P_2)} = 16.00$$

$$\frac{\partial C_2}{\partial P_2} \left(\frac{1}{1 - \frac{\partial P_L}{\partial P_2}} \right) = \frac{8 + 18 \times 10^{-3} P_2}{1 - (6 \times 10^{-5} P_2 + 2 \times 10^{-5} P_1)} = 16.00$$

Rearranging the two equation:

$$\begin{aligned} 20.8 \times 10^{-3} P_1 + 32 \times 10^{-5} P_2 &= 6.00 \\ 32 \times 10^{-5} P_1 + 18.96 \times 10^{-3} P_2 &= 8.00 \end{aligned}$$

The results will be: $P_1 = 282 \text{ MW}$, $P_2 = 417 \text{ MW}$

Substitution of the values in the problem loss model:

$$P_L = 1.5 \times 10^{-4} P_1^2 + 2 \times 10^{-5} P_1 P_2 + 3 \times 10^{-5} P_2^2 = 19.5 \text{ MW}$$

Total load demand according to Equation 3.7 will be:

$$P_D = \sum_{i=1}^{n_g} P_i - P_L = P_1 + P_2 - P_L = 282 + 417 - 19.5 = 679.5 \text{ MW}$$

Total operating cost is:

$$C_T = C_1 + C_2 = 10(282) + 8 \times 10^{-3}(282)^2 + 8(417) + 9 \times 10^{-3}(417)^2 = 8357 \text{ $/h}$$

It is worth mentioning that this example has been solved for the case which the exact value of λ is provided by the given information. The solution should be modified with the same approach as explained in the previous example, where the λ is not known but the total demanded load is given by the information. As explained earlier, in this case, the generators' inequality constraints will be considered to find the common λ . If one or more units reach their limit values, then these units are held at their limits and the remaining units operate at equal incremental fuel cost λ . The

common λ for the units that are not at their limits will be used as the incremental fuel cost of the area. Knowing the λ , the power output of each unit will be determined and consequently, the total transmission losses will be calculated according to the given loss model.

3.2.4 Derivation of Loss Formula

The system loss can be calculated according to loss coefficient or B-coefficient methods, developed by Kron and adopted by Kirchmayer [1]. This section explains the algorithm of B-coefficients and includes a review of a case study.

Assuming the total transmission losses over all buses is:

$$P_L + Q_L = \sum_{i=1}^n V_i I_i^* = \sum_{i=1}^n \sum_{j=1}^n I_i Z_{ij} I_i^* \quad (3.17)$$

$$Z_{ij} = R_{ij} + jX_{ij} \quad (3.18)$$

Where P_L and Q_L are the real and reactive power loss of the system.

Z_{ij} is the impedance matrix

I_i and I_j , are the injected bus currents.

R_{ij} , is the real part of the impedance matrix

X_{ij} is the imaginary part of the bus impedance matrix

The bus impedance matrix is symmetrical, so the $Z_{ij} = Z_{ji}$ and the real power loss equation can be formed as:

$$P_L = \sum_{i=1}^n \sum_{j=1}^n I_i R_{ij} I_i^* = I_{bus}^T R_{bus} I_{bus}^* \quad (3.19)$$

Considering individual bus currents of each load buses vary as a constant complex fraction of the total load current:

$$I_{lk} = l_k I_D \quad (3.20)$$

Where

l_k , is the complex fraction,

I_{lk} , is individual bus currents,

I_D , is the total current

Assuming bus 1 to be the reference bus (slack bus), the voltage at bus 1 can be defined by :

$$V_1 = \sum_{i=1}^{n_g} Z_{1i} I_{gi} + \sum_{i=1}^{n_d} Z_{1k} I_{Lk} \quad (3.21)$$

Where:

n_g , is the number of generator buses

n_d , is the number of load buses

Substituting Equation 3.20 into Equation 3.21 results in:

$$V_1 = \sum_{i=1}^{n_g} Z_{1i} I_{gi} + I_D T \quad (3.22)$$

Where:

$$T = \sum_{k=1}^{n_d} l_k Z_{1k} \quad (3.23)$$

Assuming I_0 is the current flowing out of bus 1, with all other current set to be zero, then V_1 is:

$$V_1 = -Z_{11} I_0 \quad (3.24)$$

By substituting Equation 3.24 in Equation 3.22 and Equation 3.20, the load currents become:

$$I_{LK} = \rho_k \sum_{i=1}^{n_g} Z_{1i} I_{gi} + \rho_k Z_{11} I_0 \quad (3.25)$$

Where:

$$\rho_k = -\frac{l_k}{T} \quad (3.26)$$

Reforming the generator currents with the above relations in matrix form results in:

$$\begin{bmatrix} I_{g1} \\ I_{g1} \\ \vdots \\ I_{L1} \\ I_{L2} \\ \vdots \\ I_{Ln_d} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \\ \rho_1 Z_{11} & \rho_1 Z_{12} & \cdots & \rho_1 Z_{1n_g} & \rho_1 Z_{11} \\ \rho_2 Z_{11} & \rho_2 Z_{12} & \cdots & \rho_2 Z_{1n_g} & \rho_2 Z_{11} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \rho_k Z_{11} & \rho_k Z_{12} & \cdots & \rho_k Z_{1n_g} & \rho_k Z_{11} \end{bmatrix} \begin{bmatrix} I_{g1} \\ I_{g1} \\ \vdots \\ I_{gnLg} \\ \vdots \\ I_0 \end{bmatrix} \quad (3.27)$$

In short form:

$$I_{bus} = C I_{new} \quad (3.28)$$

Substituting in Equation 3.19, Equation 3.29 forms as:

$$P_L = I_{new}^T C^T R_{bus} C^* I_{bus}^* \quad (3.29)$$

The generator current can be formed as:

$$I_{gi} = \frac{P_{gi} - Q_{gi}}{V_i^*} = \psi_i P_{gi} \quad (3.30)$$

Where:

$$\psi_i = \frac{1 - j \frac{Q_{gi}}{P_{gi}}}{V_i^*} \quad (3.31)$$

Adding the current I_0 to the column vector current I_{gi} in Equation 3.30 results in:

$$\begin{bmatrix} I_{g1} \\ I_{g2} \\ \vdots \\ I_{gn_g} \\ I_0 \end{bmatrix} = \begin{bmatrix} \psi_1 & 0 & \dots & 0 & 0 \\ 0 & \psi_1 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & \psi_1 & 0 \\ 0 & 0 & \dots & 0 & I_0 \end{bmatrix} \begin{bmatrix} P_{g1} \\ P_{g2} \\ \vdots \\ P_{gn_g} \\ 1 \end{bmatrix} \quad (3.32)$$

or in short form

$$I_{new} = \psi P_{G1} \quad (3.33)$$

Where

$$P_{G1} = \begin{bmatrix} P_{G1} \\ P_{G2} \\ \vdots \\ P_{Gn_g} \\ 1 \end{bmatrix} \quad (3.34)$$

Substituting Equation 3.33 in Equation 3.29 forms in:

$$P_L = P_{G1}^T \psi^T C^T R_{bus} C^* \psi^* P_{G1}^* \quad (3.35)$$

The resultant matrix from equation 3.34 is in complex form so the real power loss can be derived as:

$$P_L = P_{G1}^T R[H] P_{G1}^* \quad (3.36)$$

Where:

$$H = \psi^T C^T R_{bus} C^* \psi^* \quad (3.37)$$

H is also known as Hermitian matrix and the real part of H is found by the form

$$R[H] = \frac{H + H^*}{2} \quad (3.38)$$

The above matrix can be formed as follows:

$$R[H] = \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1n_g} & B_{01}/2 \\ B_{21} & B_{22} & \cdots & B_{2n_g} & B_{02}/2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ B_{n_g1} & B_{n_g2} & \cdots & B_{n_g n_g} & B_{0n_g}/2 \\ B_{01}/2 & B_{02}/2 & \cdots & B_{0n_g}/2 & B_{00} \end{bmatrix} \quad (3.39)$$

The power loss can be calculated according to Equation 3.40

$$P_L = \begin{bmatrix} P_{g1} & P_{g2} & \cdots & P_{gn_g} \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1n_g} \\ B_{21} & B_{22} & \cdots & B_{2n_g} \\ \vdots & \vdots & \ddots & \vdots \\ B_{n_g1} & B_{n_g2} & \cdots & B_{n_g n_g} \end{bmatrix} \begin{bmatrix} P_{g1} \\ P_{g2} \\ \vdots \\ P_{gn_g} \end{bmatrix} \\ + \begin{bmatrix} P_{g1} & P_{g2} & \cdots & P_{gn_g} \end{bmatrix} \begin{bmatrix} \frac{B_{01}}{2} \\ \frac{B_{02}}{2} \\ \vdots \\ \frac{B_{0n_g}}{2} \end{bmatrix} + B_{00} \quad (3.40)$$

3.2.5 Economic Dispatch for 7-Bus Power System

The main objective of the economic dispatch for the 7-bus power system [28] is minimizing the total generators fuel costs while considering the generators limits. Figure 3.3 shows the single line diagram of the 7-bus power system. The system contains 5 generators, 5 loads, 7 buses and 11 transmission lines. The total load demanded by this system is 760MW and 130MVA. The reference bus is located at bus 7. For the base case, the total generation cost is 16935.93 \$/hr and the total transmission loss is 7.94 MW.

Using the cost coefficients of the 5 generators in this 7-bus system, the total generation operating cost function is expressed by Equation 3.41 [28]:

$$C_i(P_{gi}) = (A_i + B_i P_{gi} + C_i (P_{gi})^2) * f_c \quad (3.41)$$

Where

f_c : fuel cost (\$/Mwh)

A_i : fuel cost dependent value (Mbtu/hr)

B_i, C_i : coefficient of the cost model

$$C_1(P_{g1}) = (373.5 + 7.62P_1 + 0.0013P_1^2) * 2.04$$

$$C_2(P_{g2}) = (403.61 + 7.519P_2 + 0.0014P_2^2) * 2.061$$

$$C_4(P_{g4}) = (253.24 + 7.836P_4 + 0.0013P_4^2) * 2.093$$

$$C_6(P_{g6}) = (388.93 + 7.573P_6 + 0.0013P_6^2) * 2.139$$

$$C_7(P_{g7}) = (194.28 + 7.771P_7 + 0.0019P_7^2) * 2.574$$

$$\begin{aligned} C_T = & (373.5 + 7.62P_1 + 0.0013P_1^2) * 2.04 + (403.61 + 7.519P_2 + 0.0014P_2^2) * 2.061 \\ & + (253.24 + 7.836P_4 + 0.0013P_4^2) * 2.093 \\ & + (388.93 + 7.573P_6 + 0.0013P_6^2) * 2.139 \\ & + (194.28 + 7.771P_7 + 0.0019P_7^2) * 2.574 = 16459.78 \text{ ($/hr)} \end{aligned}$$

The transmission loss is calculated with B-coefficients according to Equation 3.40 and has formed the equality constraints. The power generation limits of each generator are considered as the inequality constraints. Table 3.2 provides B-coefficients for 7-bus power system solved by MATLAB.

Table 3.2: B-Coefficients for 7-Bus Power System

B =	0.0212	0.0072	0.0035	0.0002	-0.004
	0.0072	0.0087	0.0018	0.0011	-0.0037
	0.0035	0.0018	0.0167	-0.0048	-0.0072
	0.0002	0.0011	-0.0048	0.0094	-0.0022
	-0.004	-0.0037	-0.0072	-0.0022	0.0103
B0 =	0	0.0004	0.0013	-0.0009	-0.0012
B00 =	3.22E-04				

According to equation 3-15 the total power loss can be calculated:

$$P_L = [P_{g1} \quad P_{g2} \quad \cdots \quad P_{gn_g}] \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1n_g} \\ B_{21} & B_{22} & \cdots & B_{2n_g} \\ \vdots & \vdots & \ddots & \vdots \\ B_{n_g1} & B_{n_g2} & \cdots & B_{n_gn_g} \end{bmatrix} \begin{bmatrix} P_{g1} \\ P_{g2} \\ \vdots \\ P_{gn_g} \end{bmatrix} +$$

$$[P_{g1} \quad P_{g2} \quad \cdots \quad P_{gn_g}] \begin{bmatrix} B_{01}/2 \\ B_{02}/2 \\ \vdots \\ B_{0n_g}/2 \end{bmatrix} = 26.11 \text{ MW}$$

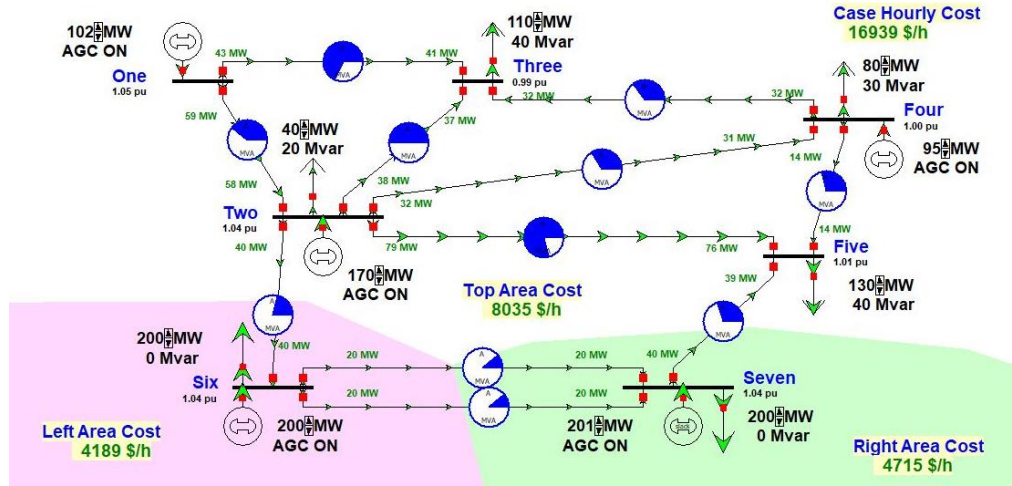


Figure 3.3: One line diagram for 7-Bus Power System [28]

Table 3.3 illustrates the results of the running economic dispatch solution with the MATLAB Optimization Toolbox and using the f-mincon function for achieving the minimum fuel cost of generation for the 7-bus power system. The total fuel cost is 16459.78 \$/hr with loss of 26.11 MW. Economic dispatch provides the system with the minimum generation fuel cost and the generation outputs are changed.

The results show that the active power output of generators has been changed and the total fuel cost has reached its minimum value.

Table 3.3: Economic Dispatch Case Study for 7-Bus System

Generation (MW)	Number	Name	Base case (MW)	Economic Dispatch by MATLAB (MW)
	1	Bus-1	101.85	100
	2	Bus-2	189.26	345.09
	4	Bus-4	95.03	102.2
	6	Bus-6	200.33	238.82
	7	Bus-7	200.65	0
Total Generation (MW)			786.11	786.11
Total Load (MW)			760	760
Total Loss (MW)			7.94	26.11
Total Hourly Cost (\$/hr)			16935.93	16459.78

3.2.6 Economic Dispatch for 37-Bus Power System

The goal of the economic dispatch power system for the 37-bus, 9 generator power system [27] is to minimize total generator fuel costs while meeting the generators real power limits and the power flow equations. Figure 3.4 shows the 37-bus power system solved by the PowerWorld Simulator [28]. The total load demanded by this system is 813.70 MW and 280.7 MVAR.

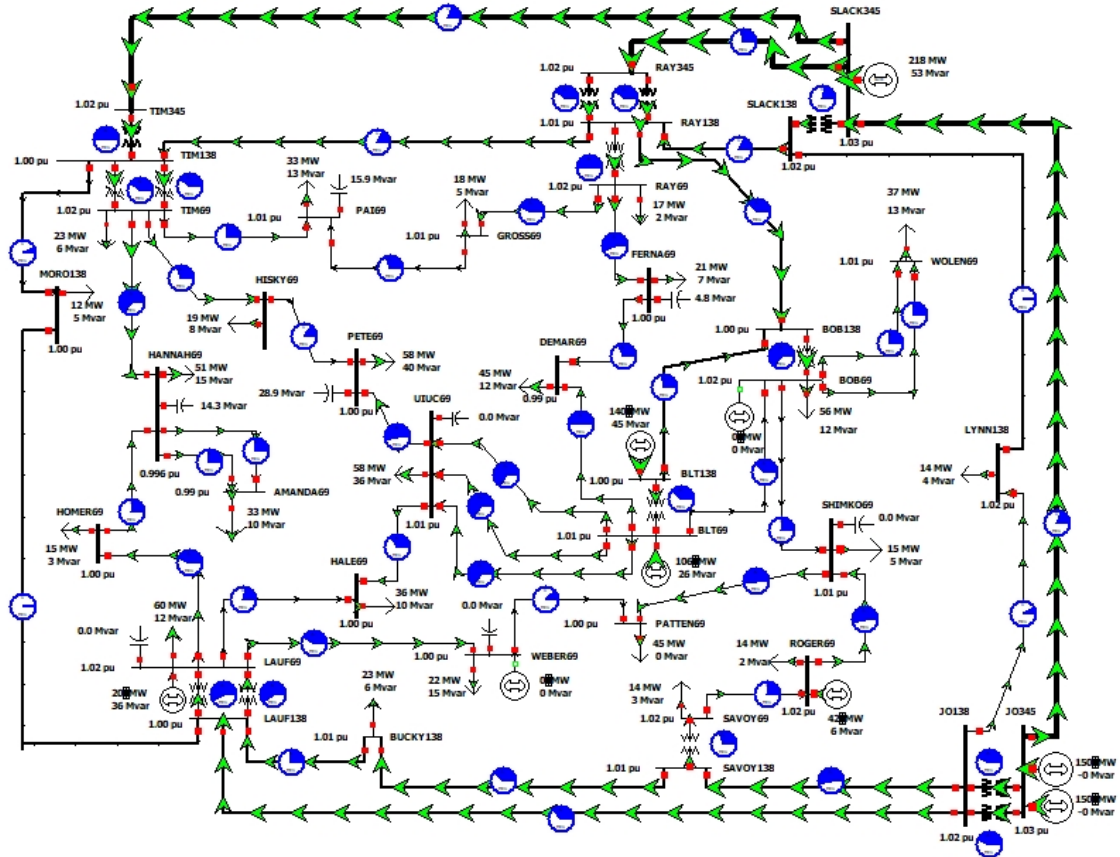


Figure 3.4: Economic Dispatch for a 37-bus Power System [29]

Table 3.3 summarizes the results of running economic dispatch solution in order to achieve the minimum fuel cost of generation for the 37-bus powers system. The results show that the active power output of generators has been changed and the total fuel cost has reached its minimum value. The software considers the transmission losses in simulation results. Economic Dispatch solution defines the most efficient approach with respect to total generation costs while it might increase the total losses or ignore some of the constraints in the system.

Table 3.4: Economic Dispatch Case Study for 37-Bus System

Generation (MW)	Bus Number	Bus Name	Base case (MW)	Economic Dispatch by PowerWorld(MW)
	14	WEBER69	10	0
	48	BOB69	16	0
	44	LAUF69	20	20
	50	ROGER69	42.09	42.09
	54	BLT69	106.78	106.08
	53	BLT138	140	140
	28	JO345	150	150
	28	JO345	150	150
	31	SLACK345	190.94	217.68
Total Generation (MW)			825.8	825.8
Total Load (MW)			813.8	813.8
Total Loss (MW)			10.79	11.59
Total Hourly Cost (\$/hr)			15848.33	15555

3.3 Optimal Power Flow

Economic dispatch ignores the limits imposed by the devices in the transmission system. This is a serious shortcoming in power generation scheduling. The power transmitted through transformers and transmission lines is restricted to lie within given limits, which arise due to thermal, voltage, or stability considerations. The combination of economic dispatch, with either the full ac power flow or a dc power flow, and enforcing the system constraints, is an efficient solution for optimizing the power generation. The result is known as the Optimal Power Flow (OPF). There are several methods for solving the OPF problem [27].

3.3.1 OPF Formulation

The mathematical formulation of the optimal power flow problem is stated as a nonlinearly constrained optimization problem. The problem is subject to satisfaction of nonlinear equality constraints and nonlinear inequality constraints. The objective function is expressed by Equation 3.42 [3] [30].

$$\begin{aligned} & \text{Minimize } F(x, u) \\ & \text{Subject to: } g(x, u) = 0 \\ & \quad \quad \quad h(x, u) \leq 0 \end{aligned} \tag{3.42}$$

where u is the vector of control or independent variables, x is the vector of state or dependent variables. $f(x, u)$ is the set of objective functions of OPF, $g(x, u)$ is the set of inequality constraints, $h(x, u)$ is the set of equality constraints.

The OPF problem has many control variables that need to be specified and subjected to a variety of equality and inequality constraints. These constraints include limits on all control variables, power flow equations, generation/load balance, branch flow limits (MW, MVar, MVA), bus voltage limits, active/reactive reserve limits, generator MVar limits, corridor (transmission interface) limits.

The OPF state variables, also called x , are defined by Equations 3.43 and the control variables, u , are defined by the Equation 3.44 [30] [31].

$$x^T = [P_{SG}, V_{L_1} \dots V_{L_{NL}}, Q_{G_1} \dots Q_{G_{NG}}, S_{l_1} \dots S_{l_{nl}}] \quad (3.43)$$

$$u^T = [P_{G_2} \dots P_{G_{NG}}, V_{G_1} \dots V_{G_{NG}}, Q_{C_1} \dots Q_{C_{NC}}, T_1 \dots T_{NT}] \quad (3.44)$$

Where

P_G : Active power generated at the PV buses

V_G : Voltage magnitude at PV buses

Q_C : Reactive power supplied by all shunt reactors

T_{NT} : Transformer load tap changer magnitude

P_{SG} : Active power of the slack bus

V_L : Voltage magnitude at PQ buses (load buses)

Q_G : Reactive power of all generator units

S_l : Transmission Line loading (Load flow)

NL : Number of load buses

NG : Number of generator buses (PV buses)

NC : Number of shunt VAR compensators

NT : Number of transformers

nl : Number of transmission lines

3.3.2 The Objective Functions

Six objective functions have been considered for optimizing the OPF problem. These objective functions vary from fuel cost generation, active and/or reactive power transmission loss, reactive power reserve margin, security margin index, and environmental emission. Some of these objective functions are conflicting in nature, which makes the OPF problem complicated [30]. A multi-objective concept will be discussed in chapter five to solve the OPF problem.

3.3.2.1 Minimization of Fuel Cost of Generation (FC)

The main objective function of the optimal power flow problem is minimizing the generation fuel cost of thermal units. The generation fuel cost objective function can be expressed by a quadratic function according to Equation 3.45.

$$\begin{aligned} \text{Minimize } TFC &= \sum_{k=1}^{N_G} FC_k(P_{Gk}) \\ FC_k(P_{Gk}) &= a_k P_{Gk}^2 + b_k P_{Gk} + c_k \end{aligned} \quad (3.45)$$

Where

FC : Total fuel cost generation of the power system

$FC_k(P_{Gk})$: Fuel cost function of the k^{th} generator

N_G : Number of generators including the slack generator

a_k : Quadratic cost coefficient of the k^{th} generator

b_k : Linear cost coefficient of the k^{th} generator

c_k : basic cost coefficient of the k^{th} generator

P_{Gk} : Real power output of the k^{th} generator.

3.3.2.2 Minimization of Active Power Transmission Loss

Active power transmission loss is one of the objective functions of the OPF problem. The problem goal is to minimize the total power loss in transmission lines and transformers in the network, represented by Equation 3.46 [30].

$$\text{Minimize } P_L = \sum_{k=1}^{N_G} P_{Gk} - \sum_{k=1}^{N_G} P_{Dk} \quad (3.46)$$

Where

P_L : Total RI^2 loss in the transmission lines and transformers of the network

P_{Gk} : Active power generation at k^{th} bus

P_{Dk} : Active power demand at k^{th} bus

3.3.2.3 Minimization of Reactive Power Transmission Loss

Voltage stability at load buses is particularly associated with the availability of reactive power. Based on this, minimizing reactive power losses is another important objective in optimal power flow analysis. The reactive power loss can be expressed as the difference between total generated reactive power and the total demanded reactive power in the network. Reactive power losses in a network can be minimized by the optimal scheduling of generated reactive power, transformer tap settings, and other compensating devices. In practice, the Q_{loss} is not necessarily positive. Equation 3.47 represents the formulation of reactive power transmission loss [30].

$$\text{Minimize } Q_{loss} = \sum_{k=1}^{N_G} Q_{Gk} - \sum_{k=1}^{N_G} Q_{Dk} , \quad K = 1, \dots, N_b \quad (3.47)$$

Where

Q_{loss} : Total reactive power transmission Loss in the power system

Q_{Gk} : Total reactive power generation at k^{th} bus

Q_{Dk} : Total reactive power demand at k^{th} bus

3.3.2.4 Maximization of Reactive Power Reserve Margin

Optimizing the reactive power reserves is one of the most important objectives in optimal power flow analyses. The amount of reactive power reserves can be assumed as a measure of the degree of voltage stability in a power network. With this perspective, maximizing the reactive power reserve margins and seeking to distribute reserves among the generators, are proposed in OPF studies. The objective can be achieved by minimizing the function presented by Equation 3.48.

$$\text{Minimize } F = \sum_{k=1}^{N_G} \left[\frac{Q_k^2}{Q_{k \max}} \right] \quad (3.48)$$

Where

Q_k : Reactive power of k^{th} generator

$Q_{k \max}$: Maximum reactive power of k^{th} generator

3.3.2.5 Minimization of Emission Index (EI)

The Emission or Environmental Index (EI), is an index from the viewpoint of environmental conservation. The atmospheric pollutants such as nitrogen oxides (NOx) and sulfur oxides (SOx) caused by fossil-fueled thermal units can be modeled separately. However, the OPF problem seeks to minimize the total (Ton/h) emission $E(P_G)$ of these pollutants, which can be stated by the following equations. As indicated, the amount of emissions is given as a function of the generator active power output, which is the sum of quadratic and exponential functions according to Equation 3.49 [30].

$$\begin{aligned} \text{Minimize } FE &= \sum_{k=1}^{N_G} E_k(P_{Gk}) \quad (3.49) \\ E_k(P_{Gk}) &= \sum_{k=1}^{N_G} \alpha_k P_{Gk}^2 + \beta_k P_{Gk} + \gamma_k + \omega_i e^{(\mu_k P_{Gk})} \end{aligned}$$

FE : Total emission of the power system

$E_k(P_{Gk})$: Emission function of k^{th} generator

$\alpha_k, \beta_k, \gamma_k, \omega_i$ and μ_k : Coefficients of the k^{th} generator

3.3.2.6 Maximization of Security Margin Index (SMI)

The last objective function is the Security Margin Index (SMI). The OPF problem seeks to operate all the transmission lines connected in a network to their maximum capability according to Equation 3.50 [30].

$$\text{Minimize } SMI = \sum_{k=1}^{N_G} (S_{li}^{max} - S_{li}) \quad (3.50)$$

3.3.2.7 Minimization of Voltage Deviation

Voltage deviation is a measure of voltage quality in the power network. The index of deviation is also important from the security aspect. The indicator is formulated as a cumulative deviation of voltages for all load buses (PQ buses) in the network from a nominal value of unity. It can be defined mathematically as presented by Equation 3.51 [31], [32].

$$VD = \left(\sum_{p=1}^{NL} |V_{Lp} - 1| \right) \quad (3.51)$$

V_{Lp} : voltage at load buses

3.3.3 The OPF Constraints

The optimal power flow problem has many control variables that are subjected to meet a variety of equality and inequality constraints. These control variables include generator bus voltages, active power in all generator units, switchable shunt reactors, and transformer tap positions. The OPF constraints are categorized into equality and inequality constraints.

3.3.3.1 Equality Constraints

The power flow equations require the net injection of the active and reactive power at each bus to be zero. These equations form the equality constraints of the OPF problem, reflecting the physics of the power system and its limitations and are presented by Equations 3.52 and 3.53.

Active Power Constraints:

$$P_G^k - P_D^k = V_k \sum_{j=1}^N \left[V_j [G_j^k \cos(\delta^k - \delta^j) + B_j^k \sin(\delta^k - \delta^j)] \right] \quad (3.52)$$

Reactive Power Constraints:

$$Q_G^k - Q_D^k = V_k \sum_{j=1}^N \left[V_j [G_j^k \sin(\delta^k - \delta^j) - B_j^k \cos(\delta^k - \delta^j)] \right] \quad (3.53)$$

Where $k = 1, 2, \dots, N$

V_k : Voltage magnitude at bus k^{th}

δ^k : Voltage angle at bus k^{th}

P_G^k : Active power generation at bus k^{th}

P_D^k : Active power demand at bus k^{th}

Q_G^k : Reactive power generation at bus k^{th}

Q_D^k : Reactive power demand at bus k^{th}

G_j^k : Real part of (k, j) element of the bus admittance matrix

B_j^k : Imaginary part of (k, j) element of the bus admittance matrix

3.3.3.2 Inequality Constraints

Inequality constraints are a set of constraints that represent the systems operational and security limits and boundaries.

Active and reactive power generation constraints for all units should be considered according to inequalities presented by Equation 3.54.

$$\begin{aligned} P_{Gk}^{Min} &\leq P_{Gk} \leq P_{Gk}^{Max} \\ Q_{Gk}^{Min} &\leq Q_{Gk} \leq Q_{Gk}^{Max} \end{aligned} \quad (3.54)$$

Where,

P_{Gk} : Active power generated by k^{th} generator

Q_{Gk} : Reactive power generated by k^{th} generator

P_{Gk}^{Min} : Minimum active power generated by k^{th} generator

P_{Gk}^{Max} : Maximum active power generated by k^{th} generator

Q_{Gk}^{Min} : Minimum reactive power generated by k^{th} generator

Q_{Gk}^{Max} : Maximum reactive power generated by k^{th} generator

P_{Gi}^{Max} : Maximum MW generation by i^{th} generator

P_{Gi}^{Min} : Minimum MW generation by i^{th} generator

Bus voltage magnitude constraints are presented by Equation 3.55.

$$V_k^{Min} \leq V_k \leq V_k^{Max} \quad (3.55)$$

Where

V_k : Voltage at k^{th} bus

V_k^{Min} : Minimum acceptable voltage at k^{th} bus

V_k^{Max} : Maximum acceptable voltage at k^{th} bus

The reactive power source capacity constraint is provided by Equation 3.56.

$$Q_{ck}^{Min} \leq Q_{ck} \leq Q_{ck}^{Max} \quad (3.56)$$

The transformer tap position constraints are represented by Equation 3.57.

$$T_k^{Min} \leq T_k \leq T_k^{Max} \quad (3.57)$$

The line thermal limit constraints for all transmission lines are according to Equation 3.58.

$$|S_i| \leq S_i^{max} \quad (3.58)$$

3.3.4 OPF Solution Methodologies

A first comprehensive survey regarding optimal power dispatch was given by H.H. Happ [33]. The solution methodologies of OPF can be broadly classified into two groups; conventional (classical) methods and intelligent methods.

Traditionally, conventional methods are effectively used to solve the OPF problem. Classical methods are based on mathematical programming techniques and can meet the requirements of objective functions, type of application, and nature of constraints. Mathematical formulations have to be simplified to find the solutions because of the extremely limited capability to solve real-world large-scale power system problems. Classical methods are weak in handling qualitative constraints. They have poor convergence, they may get stuck at local optimum, they can find only a single optimized solution in a single simulation run, they become too slow if the

number of variables is large and they are computationally expensive for the solution of a large system [26].

To overcome the shortcomings of conventional methods, intelligent methods based on Artificial Intelligence (AI) have been developed in the recent past. The major advantages of the AI methods are their relative versatility in handling various qualitative constraints, as well as their ability to find multiple optimal solutions in a single simulation run. Therefore, they are quite suitable for solving multi-objective optimization problems. In most cases, they can converge to the global optimum solution at a fast rate. The other advantages of intelligent techniques are their ability to learn and their ability to appropriately solve non-linear modeling problems. On the other hand, some drawbacks, include large dimensionality and the choice of training methodology [26]. Figure 3.5 summarizes two classifications of OPF solution methodologies.

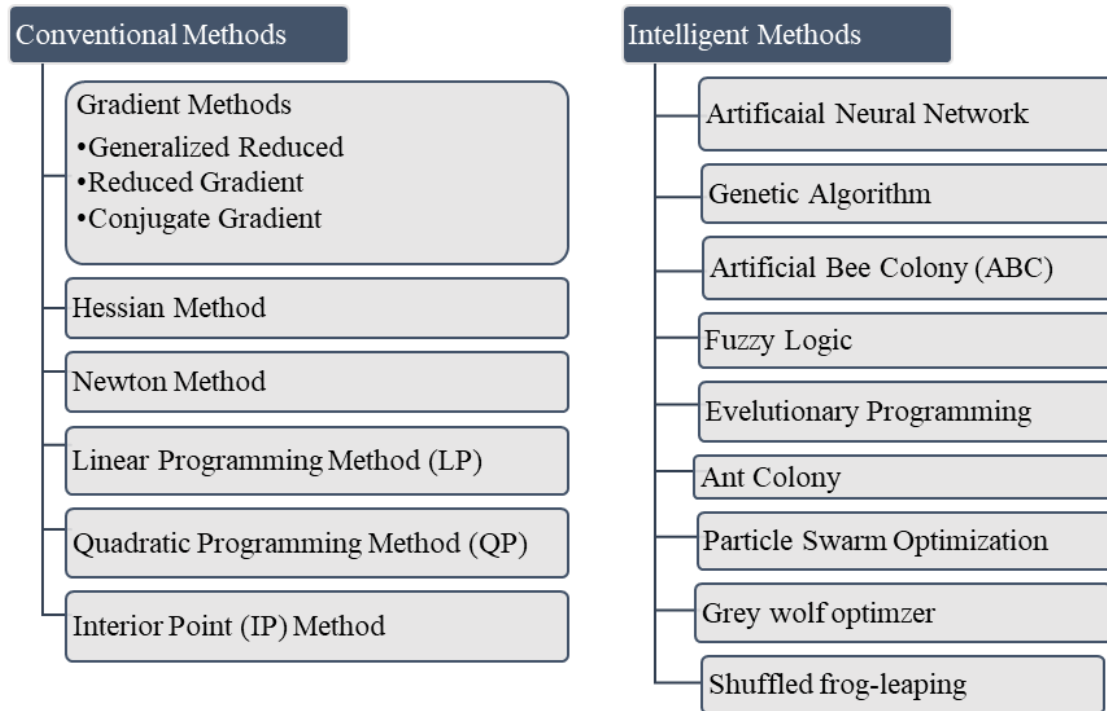


Figure 3.5: OPF solution methodologies [34]

3.3.5 Case Study of Minimizing Cost for 7-Bus Power System

The goal of OPF for the 7-bus power system case is to minimize generator fuel cost function while satisfying power flow equations, bus voltage magnitudes, the slack generator active power limits and specified branch flow limits (MVA).

Figure 3.6 shows the single line diagram of the 7-bus power system and OPF results by PowerWorld [28]. The total load is 760MW and 130 MVAR. For the base case, the total fuel cost is 16939\$/hr and transmission loss is 7.94MW.

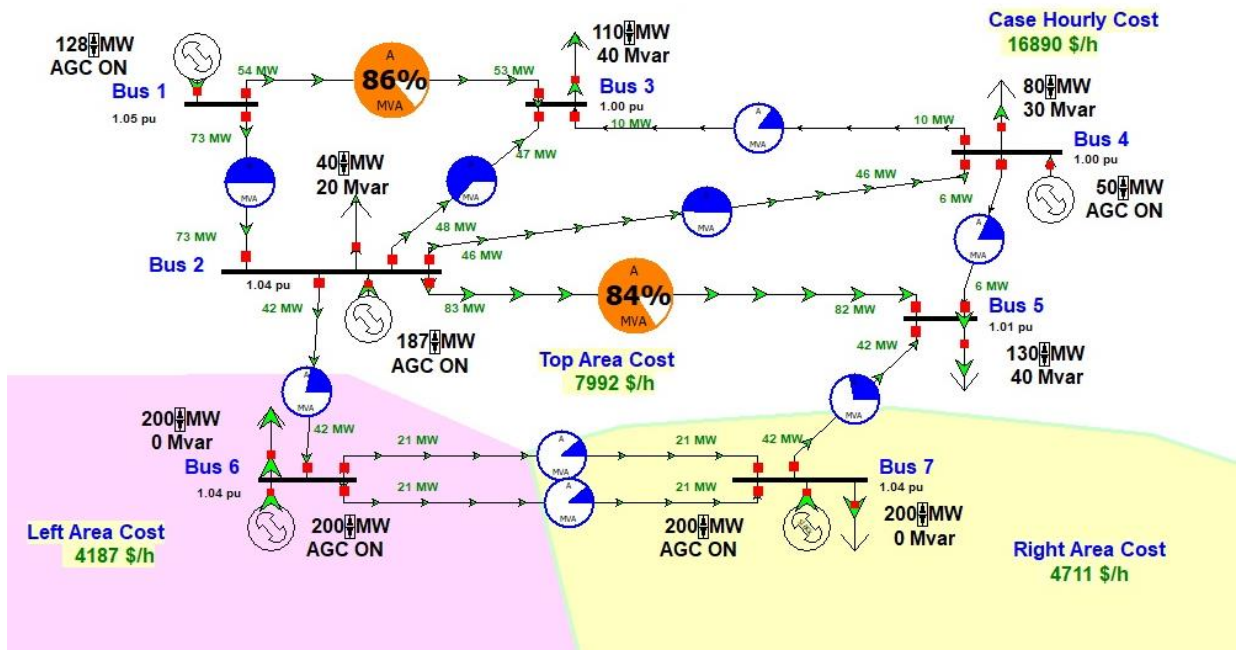


Figure 3.6: One Line Diagram of Optimal Power Flow for 7-Bus Power System with OPF [28]

The 7-bus power system contains 12 unknown variables including 4 controllable variables and 8 dependent variables. The controllable variables are four generators active power output. The dependent variables are the two bus voltage magnitudes and six phase angles. MATLAB Optimization Toolbox and the f-mincon function is applied to achieve the minimum generation cost of this power system.

Table 3.5 summarizes the results of the optimal power flow solution for the 7-bus power system without considering the security constraints. Power and loss are in MW, and the cost is in \$/hr.

The total hourly cost results for the 7-bus case is achieved 16978.74 \$/hr, using MATLAB Optimization Toolbox, and the f-mincon function. The results of running the case in PowerWorld Simulator are presented.

Table 3.5: Optimal Power Flow Case Study for 7-Bus System

Generation (MW)	Number	Name	Base case	OPF by PowerWorld	OPF by MATLAB
	1	Bus-1	101.85	127.54	127.48
	2	Bus-2	170.08	187.19	187.1
	4	Bus-4	95.03	50	50
	6	Bus-6	200.33	200.17	200.17
	7	Bus-7	200.65	200.49	205.5
Total Generation (MW)			767.94	765.39	770.25
Total Load (MW)			760	760	760
Total Loss (MW)			7.94	5.24	10.25
Total Hourly Cost (\$/hr)			16935.93	16890	16978.74

Table 3.6 compares the results of minimizing the total fuel cost function with economic dispatch and OPF methods for the 7-bus power system. Economic dispatch has the lowest cost and the base case has the highest cost. The table also illustrates that the cost of the OPF is less than the base case but it is more expensive than the results of economic dispatch. While considering the transmission lines limits, OPF methods are preferred and will prevent the overloading of the line capacity.

Table 3.6: OPF of 7-Bus Power System, Minimizing Generation Cost

Generation (MW)	Bus Name	Base Case (MW)	Economic Dispatch (MW)	OPF(MW)
	Bus-1	101.85	100	127.48
	Bus-2	170.08	345.09	187.1
	Bus-4	95.03	102.2	50
	Bus-6	200.33	238.82	200.17
	Bus-7	200.65	0	205.5
Total Hourly Cost (\$/hr)		16935.93(\$/hr)	16459.78(\$/hr)	16978.74(\$/hr)

3.3.6 Case Study of Minimizing Cost for 37-Bus Power System

The goal of the OPF for the 37-bus, 9 generator power system [27] [28] is to minimize total generator fuel costs while satisfying all the power flow constraints. The power plant includes 9 generators, 26 loads, 37 buses and 43 branches. Bus 31 is the slack bus. Figure 3.7 shows the single line diagram of the 37-bus power system solved by the PowerWorld Simulator [27].

The total load is 813.64 MW and 280.7 MVAR. The base case total fuel cost is 15848.33 \$/hr and the transmission loss is 10.79 MW.

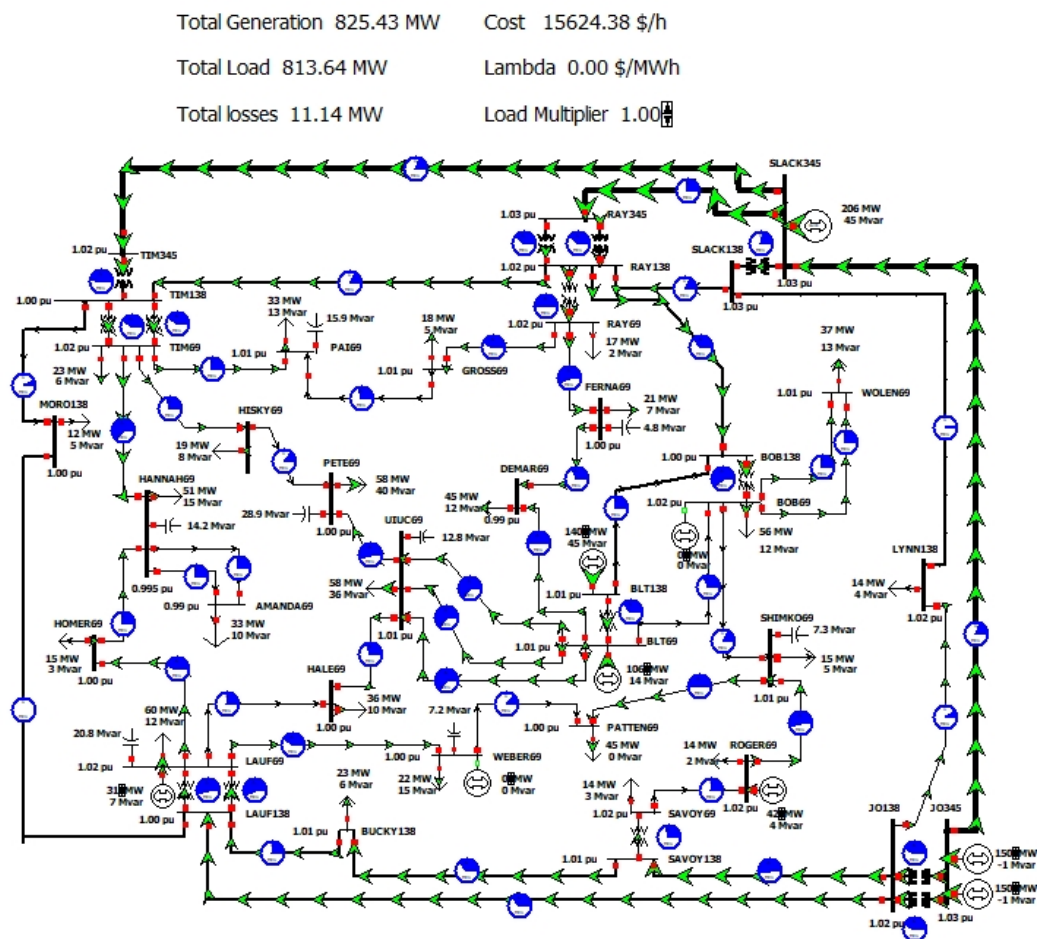


Figure 3.7: One Line Diagram of 37-Bus Power System

Table 3.7 summarizes the results of running optimal power flow by the PowerWorld Simulator in order to minimize the fuel cost of generation for 37-bus powers system.

Table 3.7: Optimal Power Flow Case Study for 37-Bus System

Generation (MW)	Bus Number	Bus Name	Base Case (MW)	Optimal Power Flow by PowerWorld (MW)
	14	WEBER69	10	0
	48	BOB69	16	150
	44	LAUF69	20	150
	50	ROGER69	42.09	205.96
	54	BLT69	106.08	31.31
	53	BLT138	140	0
	28	JO345	150	42.09
	28	JO345	150	140
	31	SLACK345	190.94	106.08
Total Generation (MW)			825.1	825.4
Total Load (MW)			813.7	813.6
Total Loss (MW)			10.79	11.14
Total Hourly Cost (\$/hr)			15848.33	15624

Table 3.8 compares the results of minimizing the total fuel cost function with economic dispatch and OPF methods for the 37-bus power system. Economic dispatch reaches the lowest cost (15555 \$/hr) and the base case has the highest cost (15848.33 \$/hr). The table also illustrates that the cost of the OPF is less than the base case but is more expensive than the results of the ED (15624 \$/hr).

The total generation cost in OPF will increase as a result of considering limits and constraints at each bus and transmission line.

Table 3.8: OPF of 37-Bus Power System, Minimizing Generation Cost

Generation (MW)	Bus Number	Bus Name	Base Case (MW)	ED (MW)	OPF(MW)
	14	WEBER69	10	0	0
	48	BOB69	16	0	150
	44	LAUF69	20	20	150
	50	ROGER69	42.09	42.09	205.96
	54	BLT69	106.08	106.08	31.31
	53	BLT138	140	140	0
	28	JO345	150	150	42.09
	28	JO345	150	150	140
	31	SLACK345	190.94	217.68	106.08
Total Hourly Cost (\$/hr)			15848.33(\$/hr)	15555 (\$/hr)	15624 (\$/hr)

3.4 Security-Constrained Optimal Power Flows (SCOPF)

A common objective of OPF problem is minimizing the total generation cost. Although it ensures the economic operation of power systems, it does not include contingency constraints and the solution might be insecure. Security constrained optimal power flow (SCOPF) is defined as an operating strategy to enforce (N-1) reliability criteria for the secure operating point. Any single contingency should not induce any further constraint violations and the design should avoid cascading failure of the network [35].

The inclusion of contingencies requires some pre-contingent and post-contingent control actions. Pre-contingent control actions normally consider line overloads and limitation, while post-contingent control actions are applied for reactive power control and generator outage re-dispatch [29].

Optimal power flow formulations can be modified to ensure that there are no contingency violations in any post-contingency states. The Security Constrained OPF (SCOPF) takes the form of Equation 3.59, denoted by C, with the given set of contingencies in the OPF problem [36] [35].

$$\text{Minimize } C(x, u) \quad (3.59)$$

Subject to satisfaction of nonlinear equality constraints:

$$\text{Subject to: } h(x, u) = 0$$

and nonlinear inequality constraints:

$$\begin{aligned} g(x, u) &\leq 0 \\ \forall m \in C: \\ g^m(x, u) &\leq 0 \\ h^m(x, u) &= 0, \end{aligned}$$

where g^m and h^m represent the network constraints under contingency m , and are referred to as the security (or contingency) constraints. For example, if contingency m considers a particular line being out of service, then the network model in constraints g^m would not contain that line [35].

A security assessment can be applied as a static security assessment or dynamic security assessment [37]. The study presented in this thesis will consider static security. The SCOPF starts the search for optimal operating point by running OPF with N constraints, then the contingency analysis is run to recognize the potential contingency cases. If there is no constraint violation in the network, the OPF will provide the optimal solution for SCOPF. If a security violation is induced by outages, the complete security constraint should be applied. The OPF and each of the contingency power flows should be re-executed until the OPF can satisfy all contingency constraints. The new optimal operating point ensures that the solution will be feasible for all contingencies under consideration. This will clearly show the concept of an inherent tradeoff between optimality and reliability in the network. Frequently adjusting the generation schedules and voltages set points will result in a less economic but more reliable operating solution [36] [35]. Many techniques are used to analyze the contingencies and SCOPF. PowerWorld Simulator has the contingency analysis function which provides the user with the contingency violation information.

3.4.1 Case Study of SCOPF

The power system with 37-bus has been considered for analyzing the SCOPF, adopted from [29]. The system has been slightly modified, mostly with some transmission lines enhancements and transformers limits to better improve its SCOPF performance. The goal for the SCOPF is to minimize generator fuel costs while considering the system security constraints and limitation. In this case, the SCOPF has focused on a single transmission line outage only. The one-line diagram of the system is shown in Figure 3.8, while there is no area control enforced to the system. Some selected information, such as total load, total losses, current MW generation by each generator, and its linear cost value are presented by a table in one line diagram. Considering this model as the base case, the total fuel cost is 25918\$/hr and the transmission losses are 40.98 MW. The total load is 1421MW and 478.1 MVar. Total generation is 1462.3MW and 576 MVar.

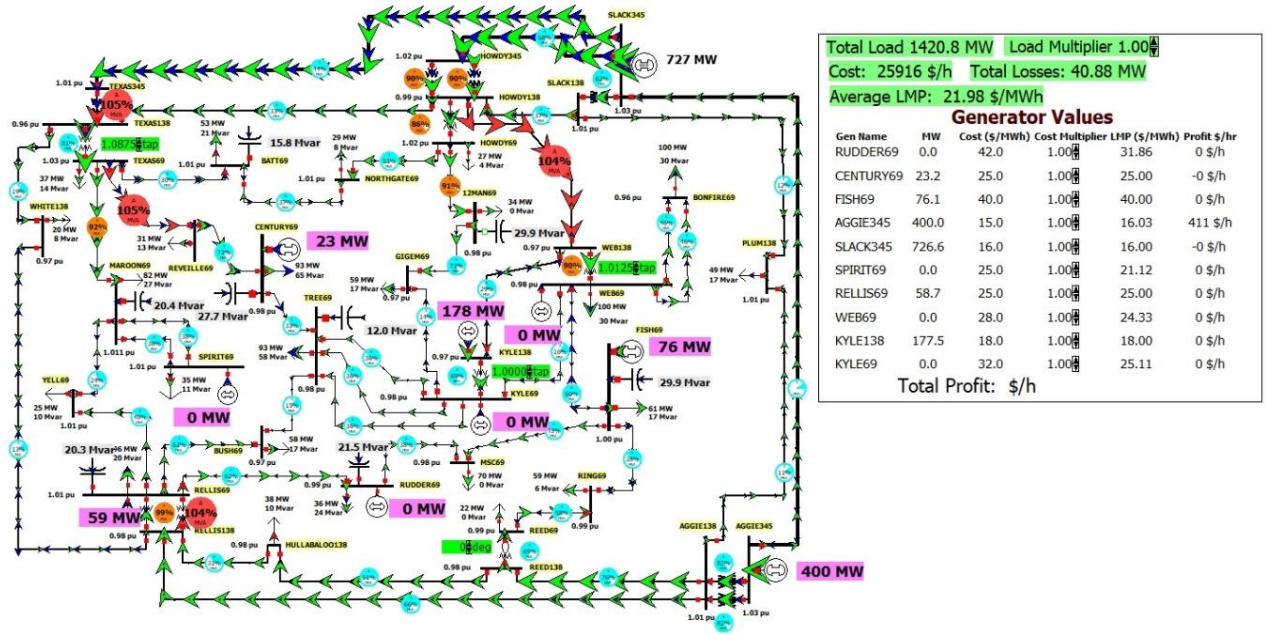


Figure 3.8: One Line Diagram of 37-Bus Power System with No Control on Area [29]

Figure 3.9 shows the same system while the area control option has been set to apply optimal power flow control in PowerWorld Simulator. The total load is approximately 1421 MW and the cost is 29752\$/h. The total loss is 22.92 (Because of tolerances in the solution, the total load, losses, and costs might change slightly by repeating a solution with the same load scalar).

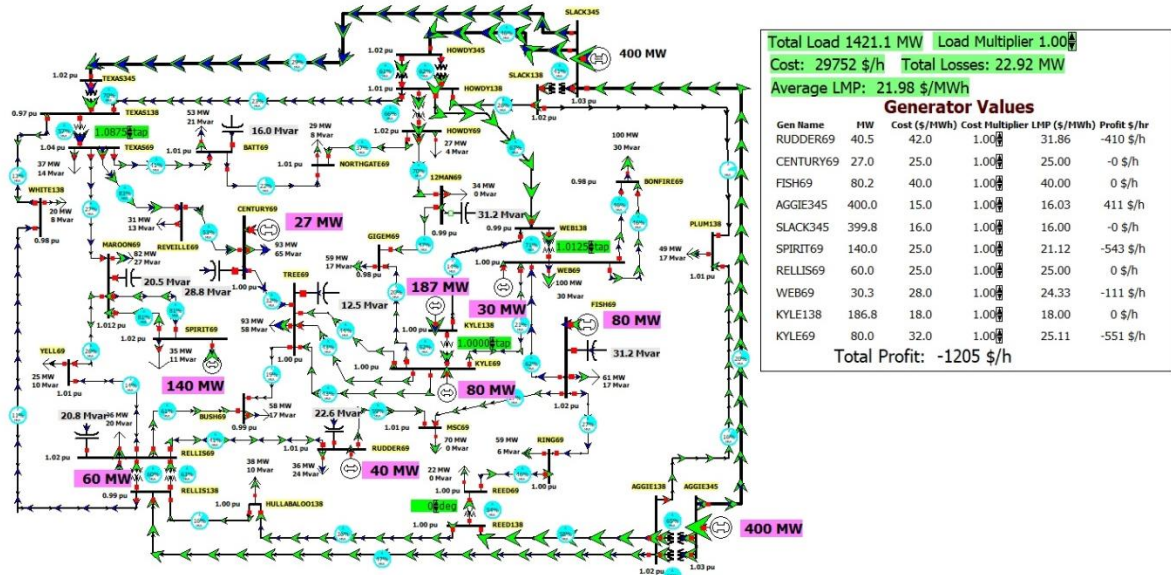


Figure 3.9: One Line Diagram of 37-Bus Power System OPF with OPF Control on Area [29]

The formulation of SCOPF can be defined according to the set of Equations 3.38. While the objective function of the study will remain the same as OPF solution, the optimization problem will consider the necessary constraints with respect to an outage of a single transmission line. Figure 3.10 shows the system while the full security constrained OPF method is performed.

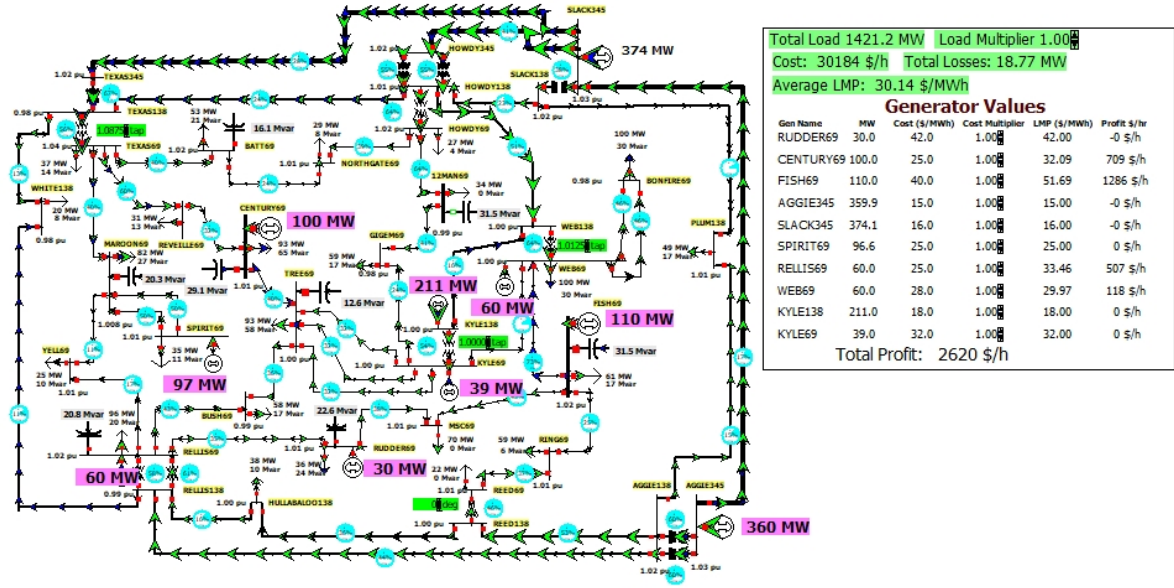


Figure 3.10: SCOPF of 37-Bus Power System [29]

Table 3.9 provides the results regarding contingency analysis and a number of violations that have been happened in each method.

Table 3.9: Contingency Analysis and Violation List

Control Option	Number of Violations
Base Case, with No Control Option	363
Optimal Power Flow Control	19
Security Control Optimal Power Flow	7

The SCOPF solution is the best possible compromise between finding an optimal solution and enforcing the constraints. It is found that in some situations it is not possible to enforce all the constraints. Otherwise, it would not be possible to find a feasible solution for the system [38].

Table 3.10 compares the output power of generators in 37-bus case study while performing the OPF and SCOPF solution.

Table 3.10: Comparing Generators Power Output in 37-Bus Power System

Number of Bus	Name of Bus	Base Case(MW)	OPF(MW)	SCOPF(MW)
14	RUDDER69	0	40.46	30.04
16	CENTURY69	23.23	26.96	100
20	FISH69	76.14	80.24	110
28	AGGIE345	400	400	359.87
31	SLACK345	726.61	399.79	374.1
37	SPIRIT69	0	140	96.57
44	RELLIS69	58.73	60	60
48	WEB69	0	30.34	60
53	KYLE138	177.53	186.84	210.95
54	KYLE69	0	80	38.98

The results of considering the outage of a single transmission line, are given in Table 3.11. Power is in MW, Voltage is in per unit and Cost is in \$/hr. The total hourly cost for SCOPF is 30184 \$/hr with the loss of 40.88 MW. The table illustrates that the output generation of each unit and total fuel cost are changed during the outage to ensure the optimal and secure operation of the system [37]. Evidently, the cost of SCOPF (30184 \$/hr) is higher than the cost of normal OPF (29752 \$/hr) which is justifiable by the significant drop in the number of system violation from 19 violations while using OPF to 7 violations while performing SCOPF. Hence the higher cost is required when the security constraints are included. All the results have been provided by PowerWorld Simulator.

Table 3.11: SCOPF of 37-Bus Power System, Minimizing Generation Cost

Area Summary Totals	Base Case		OPF		SCOPF	
	MW	Mvar	MW	Mvar	MW	Mvar
Load	1420.8	478.1	1421.1	478.1	1421.2	478.1
Generation	1462.3	566.3	1444.6	377.4	1440.5	344.1
Losses	40.88	235.91	22.92	51.81	18.77	19.01
Hourly Cost(\$/hr)	25916 (\$/hr)		29752(\$/hr)		30184 (\$/hr)	

3.5 Conclusions

This chapter has discussed the fundamentals of Economic Dispatch (ED), Optimal Power Flow (OPF), and Security Constraint Optimal Power Flow (SCOPF) and the solution to solve them. The goal of the economic dispatch problem is to minimize the total fuel cost of generation while satisfying power system limitations and constraints. The problem is subject to meet a set of equality constraints with respect to load generation balance, while as a serious shortcoming, it ignores the limits imposed by the devices in the transmission system.

The OPF, as an efficient solution to power generation scheduling, combines the economic dispatch with either the full ac power flow or a dc power flow, while enforces the system constraints. The Security Constraint Optimal Power Flow (SCOPF) includes contingency constraints analysis and ensures the secure operation of the system while having the same objective of OPF with respect to minimizing the total generation costs of the power system. It is important to highlight the fact that the optimal generation schedule with security constraints considerations will require a higher cost in comparison to other methods.

Different case studies with respect to each concept (ED, OPF and SCOPF) are presented and simulated using MATLAB and PowerWorld Simulator to explain the solution methodologies. Case studies include standard IEEE 7-bus and IEEE 37-bus power systems.

Chapter 4

4. Multi-Objective Optimization

4.0 Introduction

The primary intention of this chapter is to provide an introduction to the general concepts and principles of multi-objective optimization. Then, classifications for multi-objective optimization algorithms are presented.

Single-objective optimization corresponds to the minimum or maximum value of a single objective function. Multi-objective optimization methods receive impetus in many practical engineering fields, where the problem needs to achieve a compromise in the presence of trade-offs between two or more conflicting objective functions and find the best set of values for each one of them.

The first part of this chapter explains the theoretical concept of the single objective optimization problem and the formulation. Then, a comprehensive model of multi-objective optimization methods is presented. The final section deals with practical insights using two case studies.

4.1 Single Objective Optimization

Single-objective optimization corresponds to the minimum or maximum value of a single objective function. The problem could be formed by either an objective function or constraint functions, with linear or nonlinear, convex or non-convex, explicit or implicit characteristics.

4.1.1 Single Objective Optimization Formulation

By definition, a function of one variable $f(x)$ is said to have a relative or local minimum at $x = x^*$ if $f(x) \leq f(x^* + h)$ for all sufficiently small positive and negative values of h . Also, a point x^* is called a relative or local maximum if $f(x) \geq f(x^* + h)$ for all values of h sufficiently close to zero [4].

A function $f(x)$ is said to have a global or absolute minimum at x^* if $f(x^*) \leq f(x)$ for all x in the domain over which $f(x)$ is defined. Similarly, a point x^* will be a global maximum of $f(x)$ if $f(x^*) \geq f(x)$ for all x in the domain. Figure 4.1 shows the difference between the local and global optimum points.

A single objective optimization problem can be formulated as finding the variable $x = x^*$ in the interval $[a, b]$ such that x^* minimizes the objective function $f(x)$.

The standard form for a single-objective, non-linear, constrained optimization problem is provided in Equation 4.1 [4].

$$\begin{aligned} & \text{Minimize} && f(x) && (4.1) \\ & \text{Subject to} && g_i(x) \leq 0 && i = 1, \dots, m \\ & && h_j(x) = 0 && j = 1, \dots, \ell \\ & && x_L \leq x \leq x_U \end{aligned}$$

where $x = (x_1, x_2, \dots, x_n)^T$ is a column vector of n real-valued design variables corresponding the optimal solution. In Equation 4.1, $f(x)$ represents the objective function, $g_i(x)$ an inequality constraint and $h_j(x)$ an equality constraint function. The searchable design region is defined by the upper and lower bounds (or side constraints), x_i^L and x_i^U , of the design variables. Figure 4.1 shows the local and global optimum points for function $f(x)$.

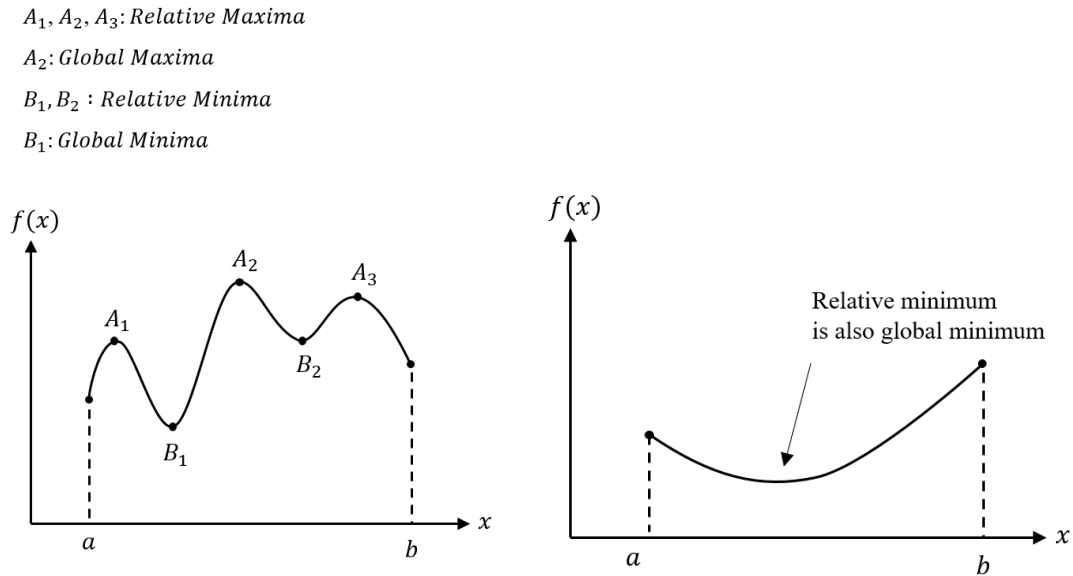


Figure 4.1: Local and Global Optimum Points [4]

4.1.2 Case Study of Single Objective Optimization

Determine the maximum and minimum values of the function

$$f(x) = 12x^5 - 45x^4 + 40x^3 + 5.$$

Solution:

Since

$$f'(x) = 60(x^4 - 3x^3 + 2x^2) = 60x^2(x - 1)(x - 2),$$

$$f'(x) = 0$$

$$\text{at } x = 0, x = 1, \text{ and } x = 2.$$

The second derivative is

$$f''(x) = 60(4x^3 - 9x^2 + 4x)$$

At $x = 1$, $f''(x) = -60$ and hence $x = 1$ is a relative maximum. Therefore,

$$f_{\max} = f(x = 1) = 12$$

At $x = 2$, $f''(x) = 240$, and hence $x = 2$ is a relative minimum. Therefore,

$$f_{\min} = f(x = 2) = -11$$

At $x = 0$, $f''(x) = 0$ and hence we must investigate the next derivative:

$$f'''(x) = 60(12x^2 - 18x + 4) = 240 \quad \text{at } x = 0$$

Since $f'''(x) \neq 0$ at $x = 0$, $x = 0$ is neither a maximum nor a minimum, and it is an inflection point [4].

4.2 Multi-Objective Optimization

The Multi-Objective Optimization Problem (MOP) involves mathematical approaches to minimize or maximize several objective functions simultaneously, considering a set of constraints. It can be used in a theoretical or practical scenario, where the decision maker seeks an optimal set of answers considering tradeoffs between conflicting objectives.

There are many applications associated with the implementation of multi-optimization techniques, including planning, programming, operation, and engineering. Consequently, these applications become a benefit to economics, business and many other fields of science, all of which try to achieve a compromise between objectives, in order to find the optimal solution.

Multi-optimization problems are characterized by having no unique ideal solution that can link competing objectives while meeting various constraints at the same time. Defining the priorities of the decision maker can lead to the best mathematical equality set of solutions. As a conceptual example of a multi-objective optimization problem, assume that we need to buy a flight ticket. The objectives in this decision would be the price of the ticket, a specific budget, the duration of the flight, the airline's service qualities, the time limitations, baggage allowance and regulations, flight insurance, etc. It is clear that there is no close link between the competing objectives in this problem, which can lead to a unique set of answers highly dependent on the decision maker's priorities.

For solving a multi-objective optimization problem, deciding the type of solution, which can be either local or global, is the initial step. This step will be generally followed by choosing a suitable mathematical method for the intended purpose. Then, objective functions and system constraints should be defined explicitly. By choosing the appropriate method, results would be found; if the results were unsatisfactory, options or start points will be changed. This cycle will continue until the best answer is achieved.

Many advanced software, such as MATLAB toolboxes, accelerate the pace of engineering and science. These technologies have been introduced to make mathematical computing more precise for this kind of application.

The general form of multi-objective optimization is formulated as finding the set of values for decision vector x according to Equation 4.2, such that:

$$\begin{aligned}
 &\text{Minimizes/Maximizes} && f_m(X) && , && m = 1, 2, \dots, M && (4.2) \\
 &\text{Subject to} && g_j(X) \geq 0 && , && j = 1, 2, \dots, J \\
 &&& h_k(X) = 0 && , && k = 1, 2, \dots, K \\
 &&& x_j^{(L)} \leq x_i \leq x_j^{(U)} && , && i = 1, 2, \dots, N
 \end{aligned}$$

$\underset{\text{Band}}{x_j^{(L)}}$
 $\underset{\text{Band}}{x_j^{(U)}}$

Equation 4.2 defined $f_m(X)$ as the objective function set (also called objective criteria, payoff functions, cost functions, or value functions); M is the number of objectives, X is a vector of design variables (also called decision variables), where N is the number of independent variables x_n ; $g_j(X)$ and $h_k(X)$ are inequality and equality constraints respectively; $x_j^{(L)}$ and $x_j^{(U)}$ are the restricting boundary conditions for decision variables.

4.3 Non-Dominated and Dominated Points, Pareto Solution

When comparing the solution of a single objective optimization problem, the objective function values will determine the superiority of one solution over another, while in a multi-objective optimization problem this concept would be explained by the definition of dominance. A vector of objective functions, $F(x^*) \in Z$, is non-dominated if there does not exist another vector, $F(x) \in Z$, such that $F(x) < F(x^*)$ with at least on $F_i(x) < F_i(x^*)$. Otherwise, $F(x^*)$ is dominated. In other words, the solution X dominates Y , if X has the same or better solution for all the objectives (equally no worse than Y in all objectives); and also the solution presented by X is strictly better than Y in at least one objective function. In this case, it could be defined that X dominates Y or equally Y is dominated by X [13]. Figure 4.2 shows the non-dominated and dominated points for function f_1 and f_2 [13].

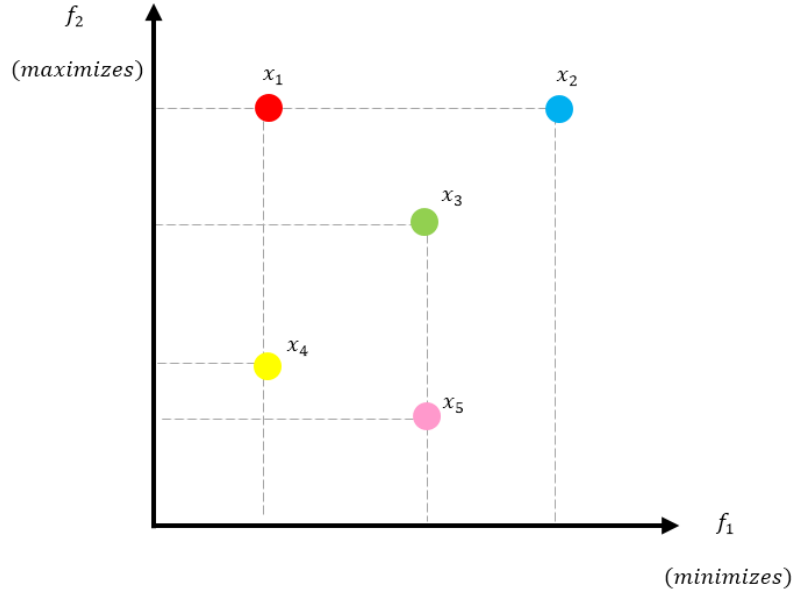


Figure 4.2: Non-dominated and dominated points [15]

- x_1 dominates x_4 , x_4 is dominated by x_1 .
- x_3 dominates x_5 , or x_5 is dominated by x_3 .
- x_3 and x_2 neither of the solutions dominates (x_3 and x_2 are not dominated with respect to each other).

Within the entire search space in a given set of solutions, the non-dominated solution set is a set of all the solutions that are not dominated by any member of the solution set. The non-dominated set of the entire feasible decision space is called the Pareto-optimal set. The boundary defined by the set of all point mapped from the Pareto optimal set is called the Pareto optimal front [13].

Figure 4.3 shows the Pareto-optimal solution. Many of the multi-objective optimization methods convert multiple objectives into a single-objective optimization function by a user-defined procedure. This is known as a scalarized problem. Achieving an appropriate function of scalarization would guarantee the solutions obtained by the Pareto optimality technique [13], [3].

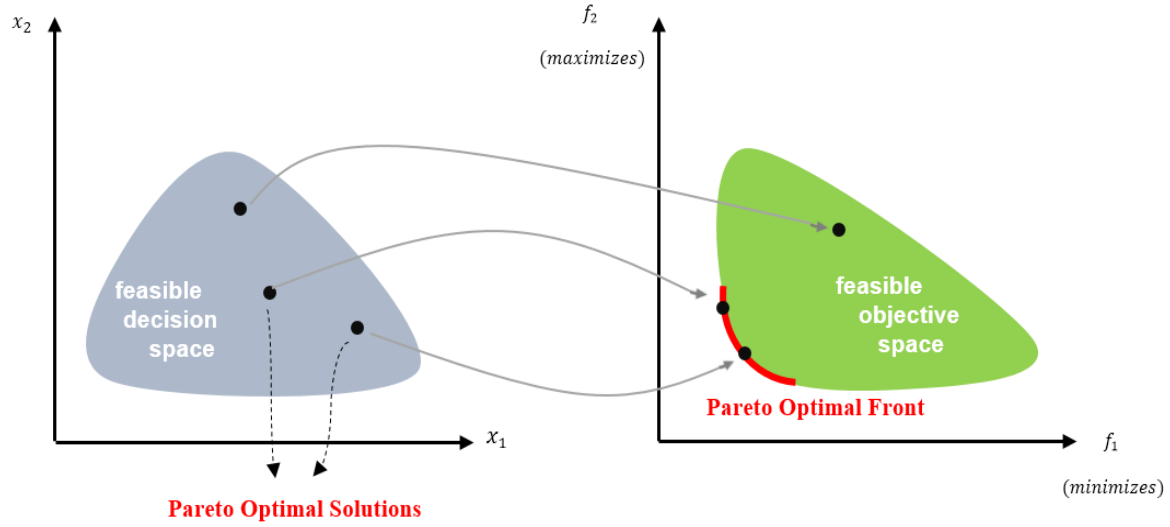


Figure 4.3: Pareto Optimal Solution [13]

The following example shows a Pareto front plot for a multi-objective function using *fgoalattain*, adopted from MATLAB tutorial [39].

Minimize: $[f_1(x) ; f_2(x)]$

where $f_1(x) = \sqrt{1 + x^2}$
 $f_2(x) = 4 + 2\sqrt{1 + (x - 1)^2}$

The two objectives in our equations are shifted and scaled versions of the convex function $\sqrt{1 + x^2}$.

In this case, it can be seen by inspection that the minimum of $f_1(x)$ is 1, and the minimum of $f_2(x)$ is 6. Figure 4.4 plots the objective functions when the variable x varies between -0.5 and 1.5, the two objectives have their minima at $x=0$ and $x=1$. Figure 4.5 represents the set of solution for Pareto optimal front.

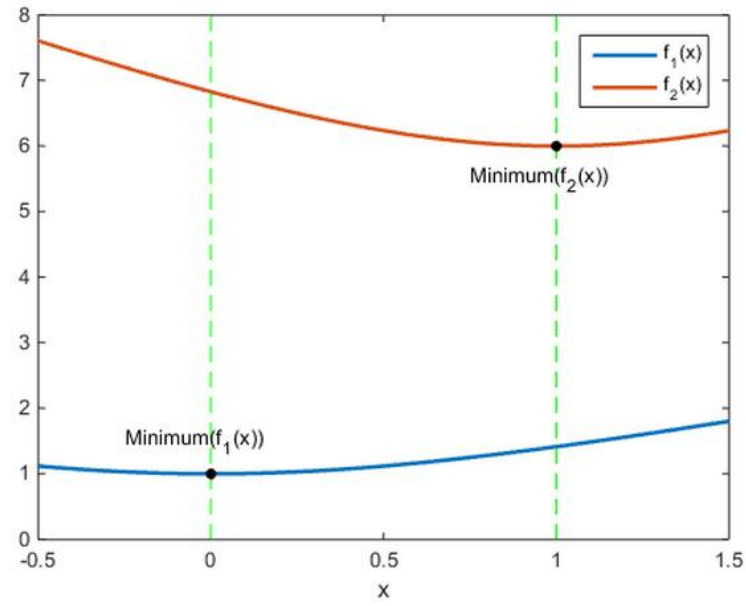


Figure 4.4: Objective Function Plot, Trade-Off Region is Between the Green Lines [12]

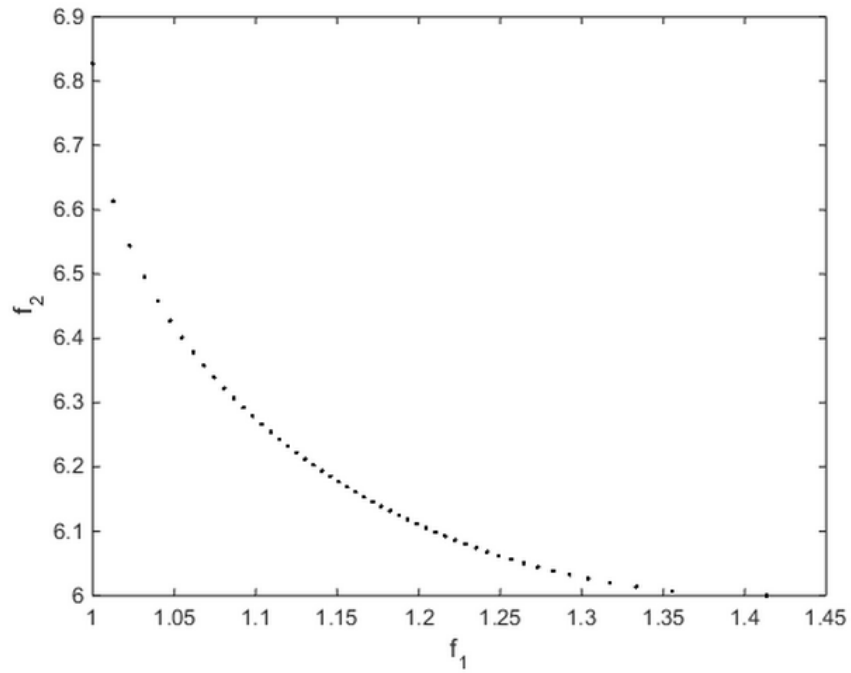


Figure 4.5: Pareto Optimal Front plot [12]

4.4 Classical Multi-Objective Optimization Methods

Classical multi-objective optimization methods have been refined for 40 years; there have been many algorithms suggested by researchers in this area, considering the classification of different methods. Cohon [40] presented the two following types of classification for Multi-Objective Optimization algorithms :

- Generating methods
- Preference-based methods

In generating methods, the decision-maker should choose one solution through the generated set of non-dominated solutions. In this case, there is no priority affecting knowledge with the final decision. While considering the preference-based methods, optimization is based on some known preference for each one of the objectives. Hwang and Masud [41] and later Miettinen [42] completed and improved the above classification according to the following classes:

- No-preference methods
- Posteriori methods
- Priori methods
- Interactive methods

In no-preference methods, a heuristic algorithm is used to generate a single optimal solution, while the objectives are not being affected by priority or importance. In posterior methods, a set of Pareto-optimal solutions would be generated iteratively using the preference knowledge of each objective. Some information about the algorithm parameter is necessary for classical methods to ensure generating Pareto optimal solutions [41] [42]. Mathematical programming and evolutionary algorithms are the two main subcategories of posterity classification.

In priori methods, sufficient preference information is expressed before the solution process. Famous applications associated with this method include the utility function method, lexicographic method, and goal programming.

Interactive methods of multi-objective optimization problems are iterative based algorithms that involve the continuous interaction of the decision maker to find the most favorable solution at each iteration in order to get Pareto optimal solution set of interest [42]. Figure 4.6 represents a classification of multi-objective optimization methods and the role of the human in the

optimization process. Figure 4.7 provides the classification of goal-oriented multi-objective optimization methods and algorithms.

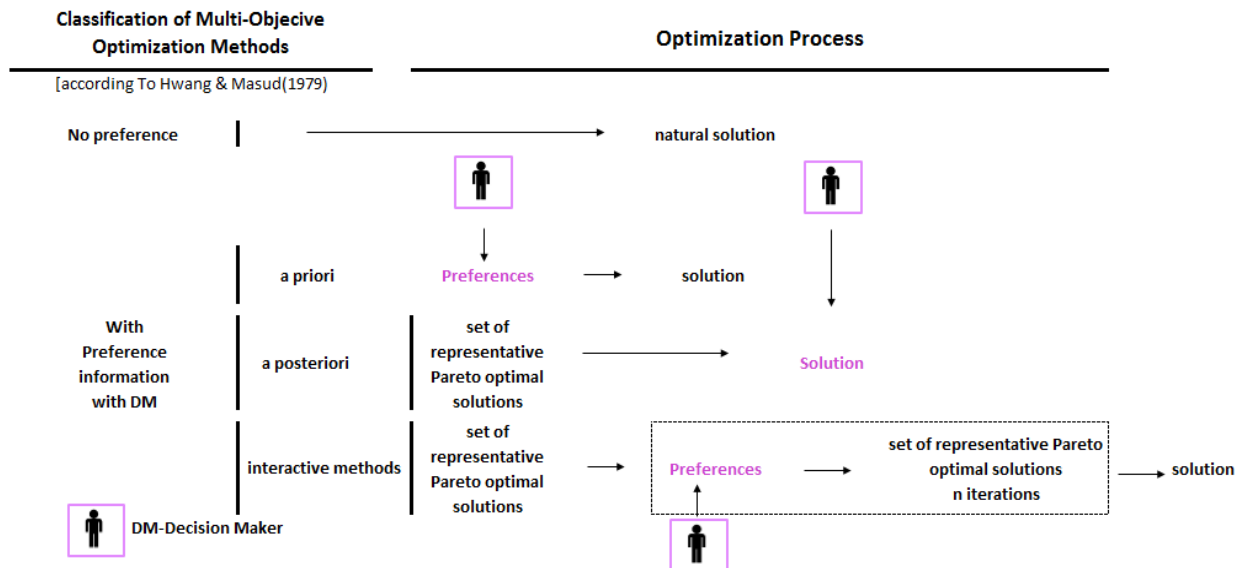


Figure 4.6: Classification of Multi-Objective Optimization Methods and the Role of Human in the Optimization Processes [43]

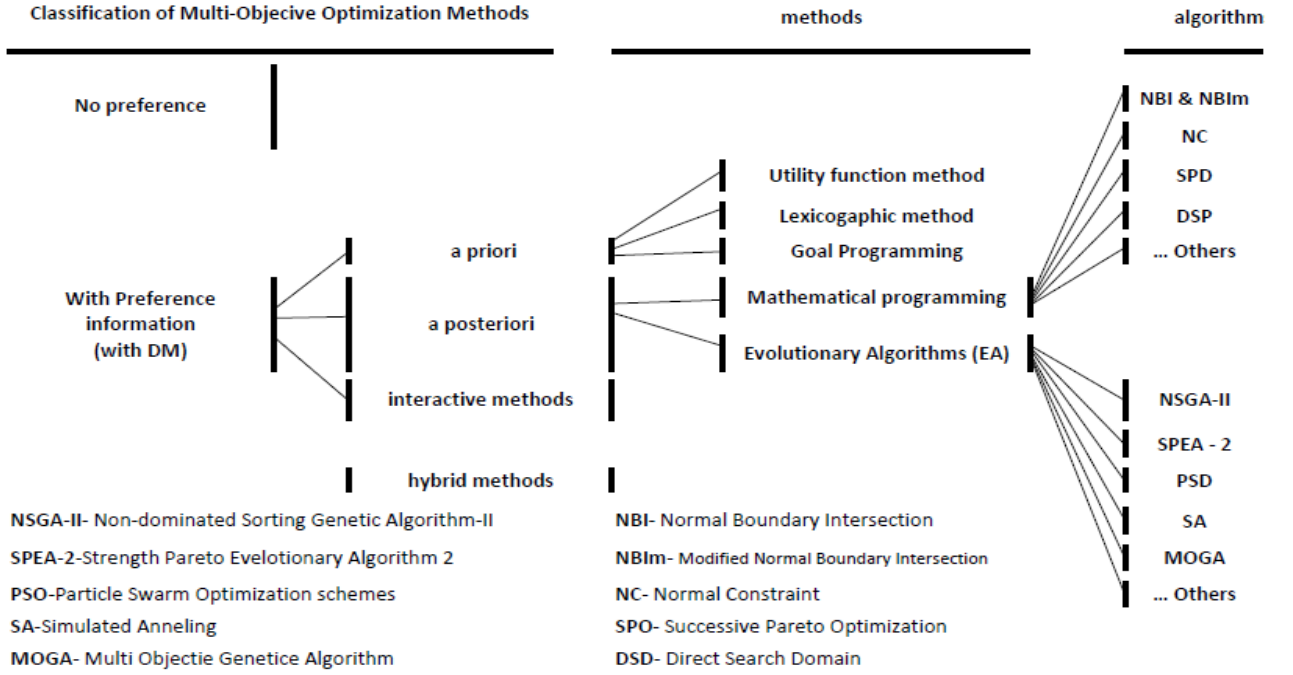


Figure 4.7: Classification of Goal-Oriented Multi-Objective Optimization Methods and Algorithms [20]

4.5 Weighted Sum Method

Weighted Sum (WS) is one of the most common classical approaches used for defining the Pareto optimal set. This method scalarizes a set of objectives into a single objective by adding each objective pre-multiplier by a user-defined weight factor. The weight of each factor would be defined by the objective relative importance in the problem [13].

$$\text{Minimizing } f(x) = \sum_{m=1}^m w_m * f_m(x), \quad m = 1, 2, \dots, M \quad (4.3)$$

$$\begin{aligned} \text{Subject to:} \quad & g_j(X) \leq 0 & j = 1, 2, \dots, J \\ & h_k(X) = 0 & k = 1, 2, \dots, K \\ & x_i^{(L)} \leq x_i \leq x_i^{(U)} & i = 1, 2, \dots, N \end{aligned}$$

In Equation (4.3), w_m is defined as the weight of the m^{th} objective function, $g_j(X)$ and $h_k(X)$ are inequality and equality constraints. $x_j^{(L)}_{Lower\ Band}$ and $x_j^{(U)}_{Upper\ Band}$ are the restricting boundary conditions for decision variables.

4.7 Case Studies of Multi-Objective Optimization

Two case studies of multi-objective optimization problems with general propose are discussed in this chapter which are adopted from MathWorks online website. The first case has two objective function and two variables, without any constraint. The second case study is a problem which consists of two objective functions, three variables, and two constraints.

4.7.1 Case Study 1

This case has a two-objective fitness function $f(x)$, where x is also two-dimensional: objective functions to be minimized are:

$$f_1 = 4x_1^6 - 2x_1^2 + 4x_1x_2 + 4x_2^6 - x_1^2x_2^2$$

$$f_2 = x_2^6 - 8x_1^2x_2^2 + x_1^6 + 2x_1x_2$$

The following function file is created before proceeding, and is stored it as mymulti1.m in MATLAB® path.

```
function f = mymulti1(x)
f(1) = 4*x(1)^6 - 2*x(1)^2 + 4*x(1)*x(2) + 4*x(2)^6 - (x(1)^2)*(x(2)^2)
f(2) = x(2)^6 - 8*(x(1)^2)*(x(2)^2) + x(1)^6 + 2*x(1)*x(2)
```

Figure 4.8 represent the MATLAB optimization toolbox setting with respect to this case study. By specifying the population size of the case for 60 for two variables, and setting the lower bound between [-5 -5] and upper bound for [5 5], the Pareto plot has been generated and represented in Figure 4.9.

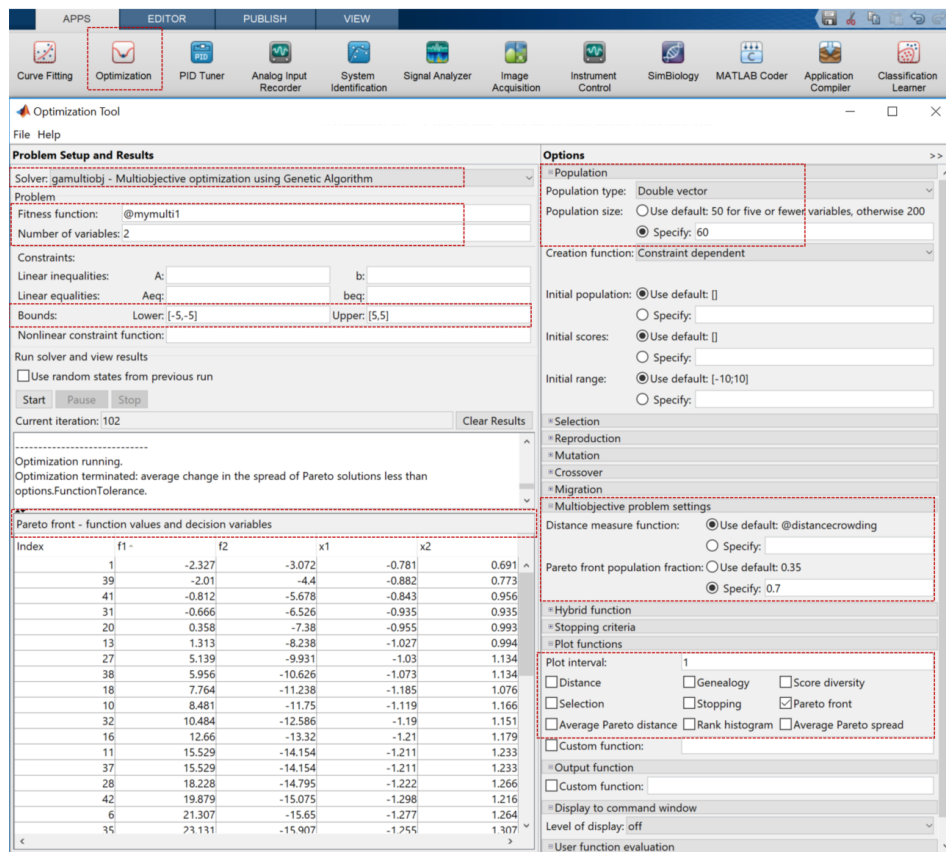


Figure 4.8: Optimization Toolbox Setting

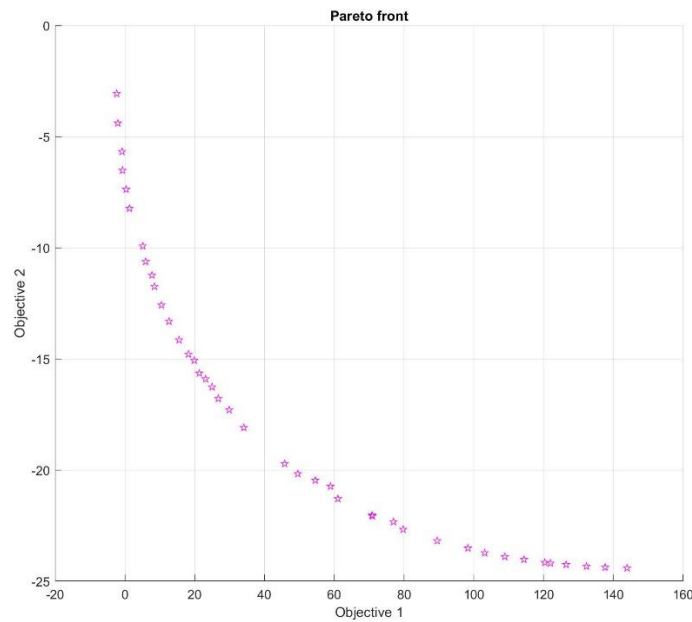


Figure 4.9: Pareto Front of Case Study 1

4.7.2 Case Study 2

In this case, there are two constraints that should be considered in two objective functions, which have to be minimized are with three variables.

$$f_1 = x_1^4 + x_2^4 + x_3^4$$

$$f_2 = x_1 x_2 (\cos x_3)^2 + x_1 x_3 (\cos x_2)^2 + x_2 x_3 (\cos x_1)^2$$

Subject to:

$$2x_1 + x_2 + x_3 = 10^5$$

$$-100 \leq x_1, x_2, x_3 \leq 100$$

By using the Weighted Sum (WS) technique, the objectives can be linearly combined as a simple objective function. Using the MATLAB optimization toolbox, by specifying the function tolerance by $1e^{-8}$, and setting the lower bound between $[-100 \ -100 \ -100]$ and upper bound for $[100 \ 100 \ 100]$, the Pareto plot has been generated and presented in Figure 4.1.

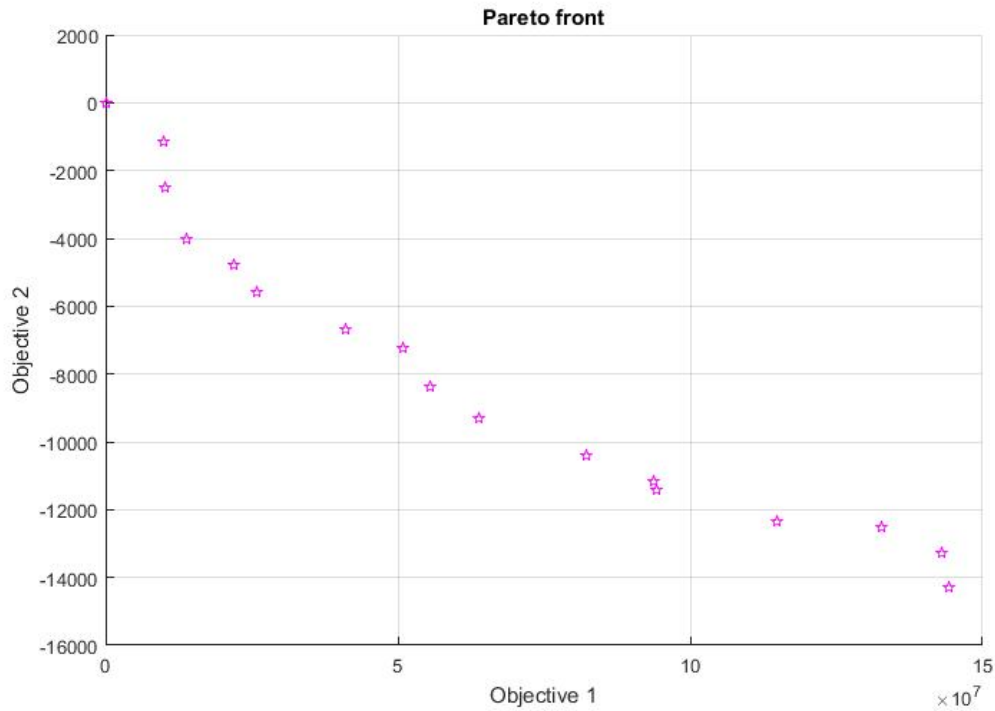


Figure 4.10: Pareto Front of Case Study 2

4.8 Conclusions

This chapter has introduced the general concepts and principles of the single objective and multi-objective optimization problems. The Pareto optimal solution has been introduced as a set of acceptable trade-off optimal solutions (non-inferior solutions). Different classifications for multi-objective optimization algorithms are presented. Weighted Sum (WS) method as one of the most common classical approaches used for defining the Pareto optimal set. Additionally, the MATLAB optimization toolbox is used for solving two numerical case studies presented in this chapter.

Chapter 5

5. Applying Multi-Objective Optimization Techniques for Solving Optimal Power Flow

5.0 Introduction

The primary propose of this chapter is to solve the Optimal Power Flow (OPF) problem as a multi-objective optimization problem while considering the system constraints. Teaching-Learning Base Optimization (TLBO) algorithm is used to optimize the OPF problem in several single and multi-objective cases. The method has been applied to 18 different cases on the IEEE 30-bus and IEEE 57-bus test systems for several OPF objectives such as minimization of fuel cost, emission, voltage deviation, and power losses. Simulation results and comparisons highlight the effectiveness of the method for solving multi-objective OPF problems and relating the results to the performance of the power system.

5.1 Multi-Objective Optimization Problem Formulation

Four objectives of the optimal power flow problem are discussed in this section. The proposed objective functions are fuel cost generation, active power transmission losses, voltage deviation, and environmental emission. It is assumed that all the generating units are thermal with a cubic cost model for the fuel expressed as a cubic function of the output of the generating units. Some of these objective functions are conflicting in nature, which makes the OPF problem complicated. A multi-objective concept will be applied to solve the OPF problem [30].

The mathematical formulation of the Optimal Power Flow problem can be stated as a nonlinear constrained optimization problem. The problem is subject to the satisfaction of nonlinear equality constraints and nonlinear inequality constraints.

The function is expressed by Equation 5.1 for the problem with two objective functions.

$$\begin{aligned}
 & \text{Minimize} \{ F_1(x, u), F_2(x, u), \dots, \text{and } F_n(x, u) \} \\
 & \text{Subject to: } g(x, u) = 0 \\
 & h(x, u) \leq 0
 \end{aligned} \tag{5.1}$$

The OPF problem has many control variables that need to be both specified and subjected to a variety of equality and inequality constraints.

In the above equation, F is the objective function to be minimized. x is the vector of dependent variables (state vector), and u is the vector of independent variables (control variables) presented in Equations 5.2:

$$\begin{aligned}
 u^T &= [P_{G2} \dots P_{G_{NG}}, V_{G1} \dots V_{G_{NG}}, Q_{C1} \dots Q_{C_{NC}}, T_1 \dots T_{NT}] \\
 x^T &= [P_{SG}, V_{L1} \dots V_{L_{NL}}, Q_{G1} \dots Q_{G_{NG}}, S_{l1} \dots S_{l_{nl}}]
 \end{aligned} \tag{5.2}$$

Where

P_G : Active power generated at the PV buses

V_G : Voltage magnitude at PV buses

Q_C : Reactive power supplied by all shunt reactors

T_{NT} : Transformer load tap changer magnitude

P_{SG} : Active power of the slack bus

V_L : Voltage magnitude at PQ buses (load buses)

Q_G : Reactive power of all generator units

S_l : Transmission Line loading (Load flow)

NL : Number of load buses

NG : Number of generator buses (PV buses)

NC : Number of shunt VAR compensators

NT : Number of transformers

nl : Number of transmission lines

The network constraints include limits on all control variables, power flow equations, generation/load balances, branch flow limits (MW, MVar, MVA), bus voltage limits, active/reactive reserve limits, generator MVar limits, corridor (transmission interface) limits. Equations 5.3 shows the equality constraints of the power system. Inequality constraints for active power constraints and reactive power constraints are provided by Equations 5.4 to 5.8.

$$P_G^k - P_D^k = V_k \sum_{j=1}^N \left[V_j [G_j^k \cos(\delta^k - \delta^j) + B_j^k \sin(\delta^k - \delta^j)] \right] \quad (5.3)$$

$$Q_G^k - Q_D^k = V_k \sum_{j=1}^N \left[V_j [G_j^k \sin(\delta^k - \delta^j) - B_j^k \cos(\delta^k - \delta^j)] \right]$$

Inequality Constraints:

$$P_{Gk}^{Min} \leq P_{Gk} \leq P_{Gk}^{Max} \quad (5.4)$$

$$Q_{Gk}^{Min} \leq Q_{Gk} \leq Q_{Gk}^{Max} \quad (5.5)$$

$$V_k^{Min} \leq V_k \leq V_k^{Max} \quad (5.6)$$

$$Q_{ck}^{Min} \leq Q_{ck} \leq Q_{ck}^{Max} \quad (5.7)$$

$$T_k^{Min} \leq T_k \leq T_k^{Max} \quad (5.8)$$

Where:

$k = 1, 2, \dots, N$

V_k : Voltage at k^{th} bus

δ^k : Voltage angle at bus k^{th}

P_G^k : Active power generation at bus k^{th}

P_D^k : Active power demand at bus k^{th}

Q_G^k : Reactive power generation at bus k^{th}

Q_D^k : Reactive power demand at bus k^{th}

G_j^k : Real part of (k, j) element of the bus admittance matrix

B_j^k : Imaginary part of (k, j) element of the bus admittance matrix

P_{Gk} : Active power generated by k^{th} generator

Q_{Gk} : Reactive power generated by k^{th} generator

Q_{ck} :The reactive power source capacity constraint

T_k : The transformer tap position constraints

It worth to mention that the control variables are self-constrained. The system generating constraints, transformer constraints, shunt VAR compensator constraints and, security constraints will satisfy the upper and lower boundaries of variables with aforementioned inequalities in solution.

The inequality constraints of state (dependent) variables can be incorporated into the objective function as a quadratic penalty term. It contains the load bus voltage magnitude, the active power of slack bus, the reactive power of generation and line loading. In these terms, a penalty factor is multiplied by with the square of the disregarded value, is added to the objective function and any unfeasible solution is declined [44]. Equation 5.9 expresses the mathematical formulation of the penalty function:

$$\begin{aligned}
 Penalty = & \lambda_P (P_{G_1} - P_{G_1}^{lim})^2 + \lambda_V \sum_{i=1}^{NL} (V_{L_i} - V_{L_i}^{Lim})^2 + \lambda_Q \sum_{i=1}^{NG} (Q_{G_i} - Q_{G_i}^{Lim})^2 \\
 & + \lambda_S \sum_{i=1}^{nl} (S_{l_i} - S_{l_i}^{Lim})^2
 \end{aligned} \tag{5.9}$$

Where $\lambda_P, \lambda_V, \lambda_Q$ and λ_S are the penalty factors and x^{lim} is the violated limit value of the state(dependent) variable x , Equation 5.10 expresses the limit value as follows:

$$x^{lim} = \begin{cases} x^{max} , & x > x^{max} \\ x^{min} , & x < x^{min} \end{cases} \tag{5.10}$$

5.2 Case Studies

Several case studies with single and multi-objectives have been performed to illustrate the effect of considering multi-objectives optimization in solving the OPF problem.

Table 5.1: Summary of case studies

Case no	IEEE 30-bus system				IEEE 57-bus system			
	Fuel Cost	Emission	Voltage Deviation	Power Losses	Fuel Cost	Emission	Voltage Deviation	Power Losses
Case 1	✓							
Case 2		✓						
Case 3			✓					
Case 4				✓				
Case 5	✓	✓						
Case 6	✓		✓					
Case 7	✓							
Case 8	✓	✓		✓				
Case 9	✓	✓	✓	✓				
Case 10					✓			
Case 11						✓		
Case 12							✓	
Case 13								✓
Case 14					✓	✓		
Case 15					✓			✓
Case 16					✓		✓	
Case 17					✓	✓		✓
Case 18					✓	✓	✓	✓

5.2.1 IEEE 30-Bus Power System

The IEEE 30-bus system is the proposed system in which the line data and the bus data are given in [31]. Different cases are considered to evaluated and investigate the basic and complex objective functions.

The system active and reactive power demands are 283.4 MW and 126.2 MW, respectively. In addition, the algorithms are implemented to solve all cases under the same conditions, i.e. system data, constraints, control variables limits. Table 5.2 provides a summary of the system specification. Bus 1 is the swing bus or slack bus. In the power flow study, the rule of the slack bus is to balance active and reactive power in the system, while satisfying power balances mentioned by Equations 5.4 and 5.5.

The first four case studies minimize the single objective functions of OPF including fuel cost, emission, transmission losses, and voltage deviation. The remaining cases are for multi-objective optimizations which are converted to single objectives with weight factors as in many previous studies [31].

Table 5.2: The Main Characteristics of The IEEE 30-Bus Test System [31]

Items	IEEE 30-bus system	
	Quantity	Details
Buses	30	Ref [31]
Branches	41	Ref [31]
Generators	6	Buses:1(Slack), 2, 5, 8 , 11 and 13
Shunt VAR compensation	9	Buses: 10, 12, 15, 17, 20, 21, 23, 24 and 29
Transformer with tab changer	4	Branches: 11, 12, 15 and 36
Control Variables	24	
Connected load		283.4 MW, 126.2 MVAr
Load bus voltage range allowed	24	[0.95- 1.05] p.u.

Table 5.3 presents the cost and emission coefficient of generators for IEEE 30-bus system.

Table 5.3: Cost and Emission Coefficient of Generators for IEEE 30-bus System [31]

Generator	Bus	a	b	c	d	e	α	β	γ	ω	μ
G1	1	0	2	0.00375	18	0.037	4.091	-5.554	6.49	0.0002	2.857
G2	2	0	1.75	0.0175	16	0.038	2.543	-6.047	5.638	0.0005	3.333
G3	5	0	1	0.0625	14	0.04	4.258	-5.094	4.586	0.000001	8
G4	8	0	3.25	0.00834	12	0.045	5.326	-3.55	3.38	0.002	2
G5	11	0	3	0.025	13	0.042	4.258	-5.094	4.586	0.000001	8
G6	13	0	3	0.025	13.5	0.041	6.131	-5.555	5.151	0.0001	6.667

Case studies one to four provides a comparison for optimizing the single objective function of OPF problem including fuel cost function, emission, the active power losses in transmission lines, and voltage deviation separately.

Case 1: Minimization of Fuel Cost

The main objective function of the optimal power flow problem is minimizing the generation Fuel Cost (FC) of all thermal units, presented by Equation 5.11. Table 5.4 summarizes the results of applying TLBO optimization technique with MATLAB on this case study.

$$FC = \sum_{i=1}^{NG} a_i P_{G_i}^2 + b_i P_{G_i} + c_i \quad (5.11)$$

Where

FC : Total fuel cost generation of the power system

NG : Number of generators including the slack generator

a_i : Quadratic cost coefficient of the k^{th} generator

b_i : Linear cost coefficient of the k^{th} generator

c_i : basic cost coefficient of the k^{th} generator

P_{G_i} : Real power output of the k^{th} generator.

Figure 5.1 shows the minimization of the fuel cost function with respect to the number of evaluation. The optimization process leads to the fuel cost of 800.7832 \$/hr while satisfying all system constraints.

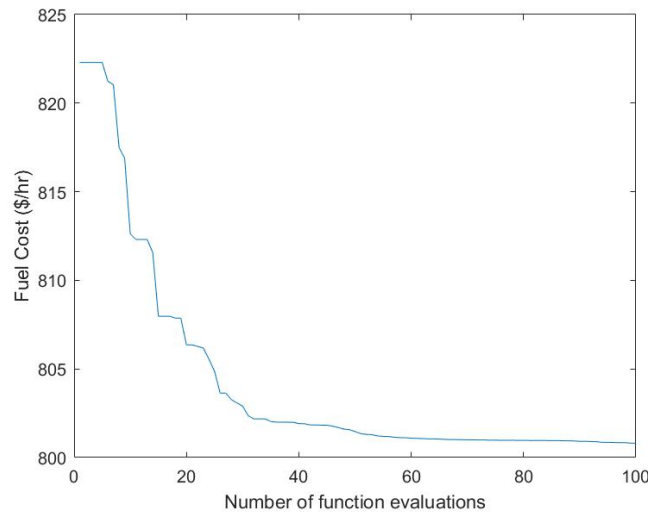


Figure 5.1: Convergence of Case 1 for IEEE 30-Bus System

Case 2: Minimization of Emission

The Emission or Environmental Index (EI), is an index from the viewpoint of environmental conservation. The atmospheric pollutants can be modeled separately. However, the OPF problem seeks to minimize the total (Ton/h) emission $E(P_G)$ of pollutants such as nitrogen oxides (NOx) and sulfur oxides (SOx) caused by fossil-fueled thermal units, which can be stated by the following equations. As indicated, the amount of emissions are given as a function of the generator active power output, which is the sum of quadratic and exponential functions according to Equation 5.12 [30].

$$F(x, u) = \text{Emission} = \sum_{k=1}^{N_G} (\alpha_i + \beta_i P_{G_i} + \gamma_i P_{G_i}^2) \times 0.01 + \omega_i e^{(\mu_i P_{G_i})} \quad (5.12)$$

Where α_i , β_i , γ_k , ω_i and μ_i are coefficients, provided in Table 5.3 for the IEEE 30-bus system. The results are presented in Table 5.4.

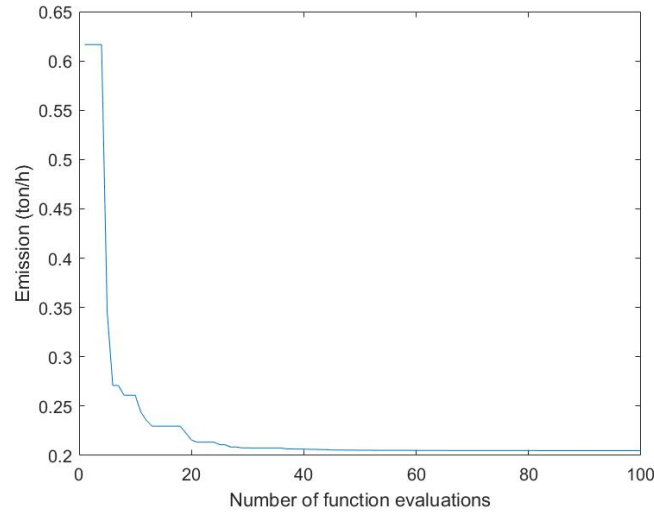


Figure 5.2: Convergence of Case 2 for IEEE30-Bus System

Figure 5.2 presents the minimization of emission function with TLBO algorithm where the optimization procedure provides the best output of 0.2049 ton/hr.

Case 3: Minimization of Voltage Deviation

Voltage deviation is a measure of voltage quality in the power network. The index of deviation is also important from the security aspect. The indicator is formulated as a cumulative deviation of voltages for all load buses (PQ buses) in the network from a nominal value of unity. It can be defined mathematically as presented by Equation 5.13 [31], [32]. Figure 5.3 illustrates the convergence of function to the optimal solution of 0.1181 p.u after 100 times evaluations by the TLBO method by MATLAB. Table 5.4 provided more details about the results of implementing the algorithm on this case study.

$$VD = \left(\sum_{P=1}^{NL} |V_{LP} - 1| \right) \quad (5.13)$$

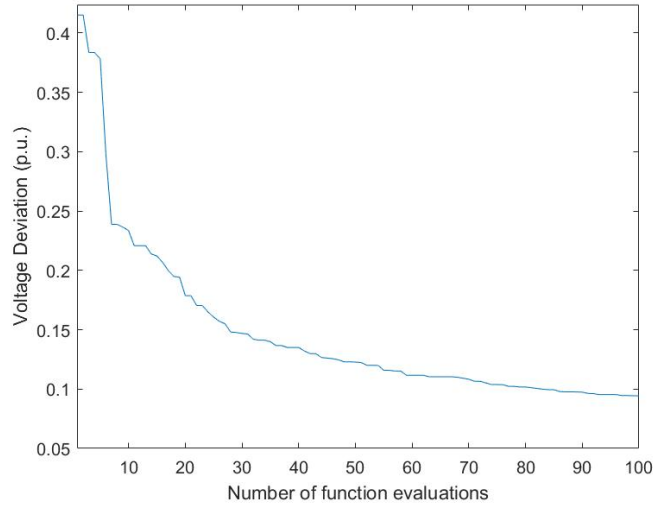


Figure 5.3: Convergence of Case 3 for IEEE 30-Bus System

Case 4: Minimization of Active Power Transmission Loss

The objective function to minimize the active power transmission loss is defined by Equation 5.14:

$$f(x, u) = P_{Loss} = \sum_{q=1}^m G_{q(ij)} [V_i^2 + V_j^2 - 2V_i V_j \cos(\delta_{ij})] \quad (5.14)$$

Where, $\delta_{ij} = \delta_i - \delta_j$, is the difference in voltage angle between bus i and bus j , $G_{q(ij)}$ is the transfer conductance of branch q connecting the buses i and j . Figure 5.4 shows the optimization procedure result in the real power loss of 3.1804 MW in transmission lines. Table 5.4 provides the details on minimization results.

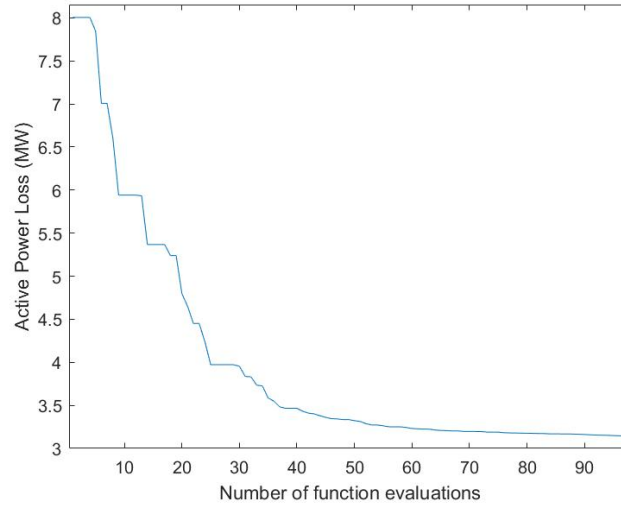


Figure 5.4: Convergence of Case 4 for IEEE 30-Bus System

Case 5: Two Objectives: Minimization of Fuel Cost and Emission

The goal of the objective function is to minimize both fuel cost and emission simultaneously. The converted single objective function will follow Equation 5.15. The weight factor λ_E is assigned the value of 1000 according to [44]. Figure 5.5 shows the convergence of the fitness function. For clarity, the convergence of each individual objective attaining the best fitness value is included in the fitness diagram for all multi-objective case studies. Table 5.5 presents the optimized solution for this multi-objective optimization problem. The best fuel cost value arrives at an increase in cost for Case 1, with the final value being 836.7029 \$/hr and the ultimate value for emission is 0.2423 ton/hr after 100 evaluations.

$$f(x, u) = \left(\sum_{i=1}^{N_G} a_i + b_i P_{G_i} + c_i P_{G_i}^2 \right) + \lambda_E \times Emission + Penalty \quad (5.15)$$

$$Emission = \sum_{k=1}^{N_G} (\alpha_i + \beta_i P_{G_i} + \gamma_i P_{G_i}^2) \times 0.01 + \omega_i e^{(\mu_i P_{G_i})}$$

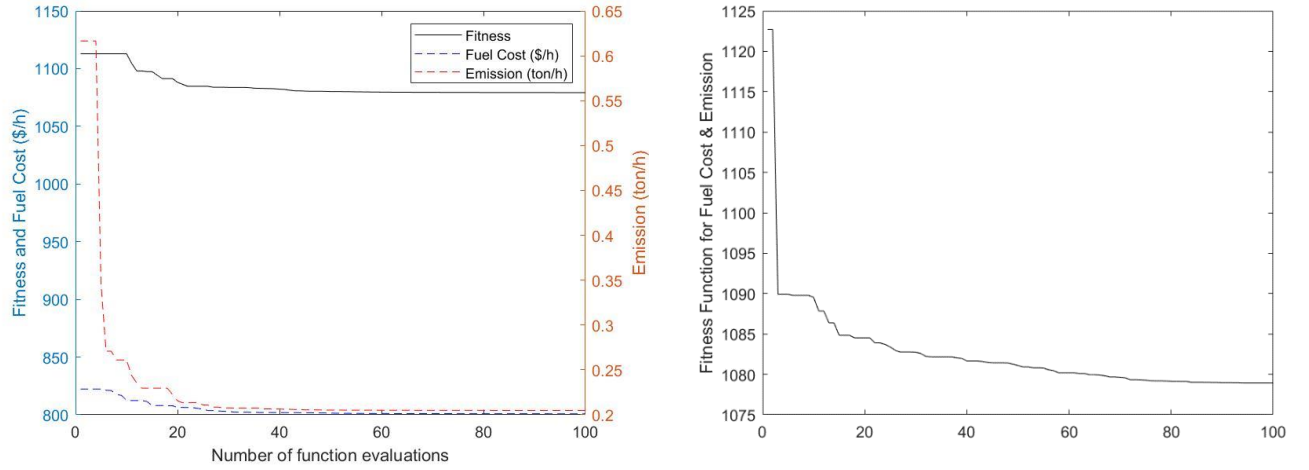


Figure 5.5: Convergence of Case 5 for IEEE 30-Bus System

Case 6: Two Objectives: Minimization of Fuel Cost and Voltage Deviation

The combined objective function of the fuel cost and the voltage deviation in the network is expressed by Equation 5.16:

$$f(x, u) = \left(\sum_{i=1}^{NG} a_i + b_i P_{G_i} + c_i P_{G_i}^2 \right) + \lambda_{VD} \times VD + Penalty \quad (5.16)$$

$$VD = \left(\sum_{P=1}^{NL} |V_{L_P} - 1| \right)$$

where the weight factor λ_{VD} is assigned a value of 100 as presented in [32] and [31]. Table 5.5 summarizes the results of implementing multi-objective optimization using TLBO algorithm using MATLAB and Figure 5.6 presents the convergence of fitness function in this case study.

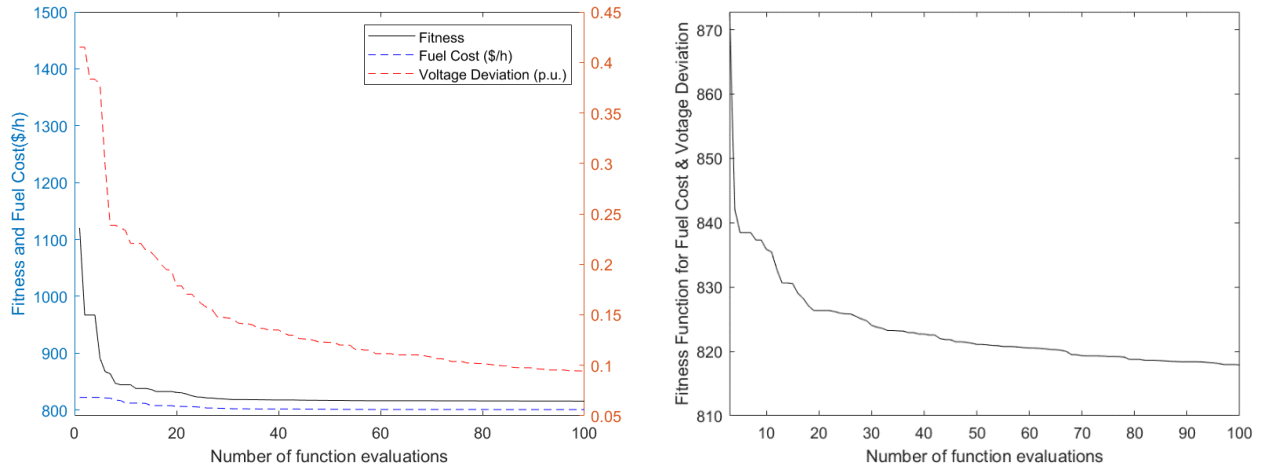


Figure 5.6: Convergence of Case 6 for IEEE30-Bus System

Case 7: Two Objectives: Minimization of Fuel Cost and Real Power Loss

The multi-objective case of minimizing two objectives of fuel cost and real power loss is converted to a single objective function by considering a multiplier as the weight factor to one of the objectives, expressed by Equation 5.17:

$$f(x, u) = \left(\sum_{i=1}^{NG} a_i + b_i P_{G_i} + c_i P_{G_i}^2 \right) + \lambda_p \times P_{Loss} + Penalty \quad (5.17)$$

$$P_{Loss} = \sum_{q=1}^m G_{q(ij)} [V_i^2 + V_j^2 - 2V_i V_j - 2V_i V_j \cos(\delta_{ij})]$$

Where the value of λ_p is chosen as 40 according to [32], and P_{Loss} is the real power loss in the network, $\delta_{ij} = \delta_i - \delta_j$, is the difference in voltage angle between bus i and bus j , and $G_{q(ij)}$ is the transfer conductance of branch q connecting the buses i and j . Figure 5.7 shows the convergence of fitness function for the multi-objective optimization of OPF where the fuel cost reaches 836.7029 \$/hr and real power loss. Table 5.5 summarizes the results of implementing the TLBO algorithm by MATLAB on this case study [31].

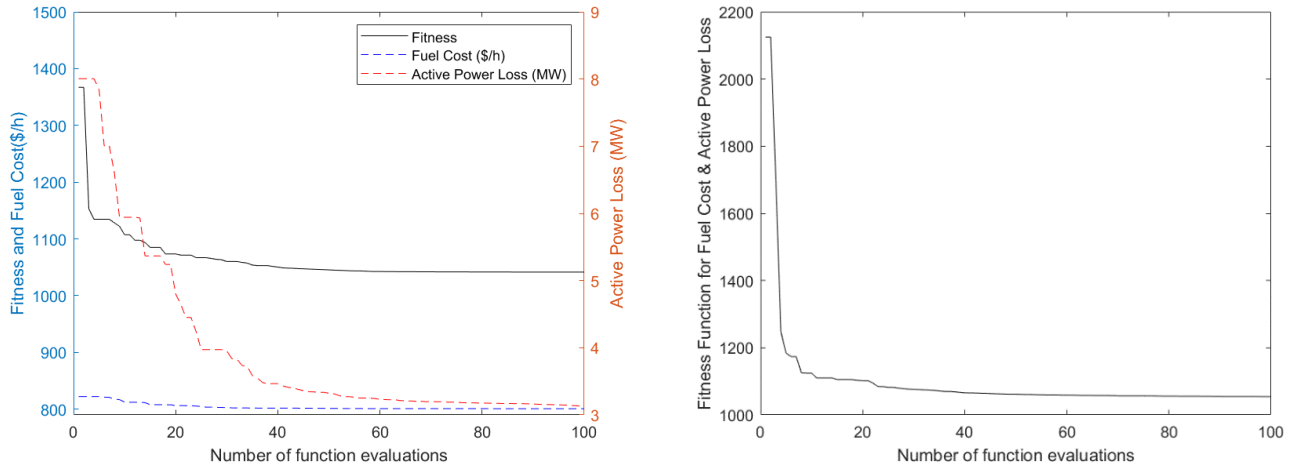


Figure 5.7: Convergence of Case 7 for IEEE30-Bus System

Case 8: Three Objectives: Minimization of Fuel Cost, Emission, Losses

The multi-objective case of minimizing fuel cost, emission, and real power loss is converted to a single objective function by multiplying a weight factor to the objectives according to Equation 5.18.

$$f(x, u) = \left(\sum_{i=1}^{NG} a_i + b_i P_{G_i} + c_i P_{G_i}^2 \right) + \lambda_E \times Emission + \lambda_P \times P_{Loss} + Penalty \quad (5.18)$$

Where emission and real power loss is expressed according to Equations 5.12 and 5.14 respectively. The weight factors are selected to balance between the objectives as 19 and 22, for emission and losses, respectively [32]. Figure 5.8 illustrates the convergence of fitness function and Table 5.5 shows the optimized values with respect to this multi-objective optimization case. The fitness function convergence revealed the best values of the function to be 830.6375 \$/hr, 0.2529 ton/hr, 5.6193 MW for fuel cost, emission and losses respectively.

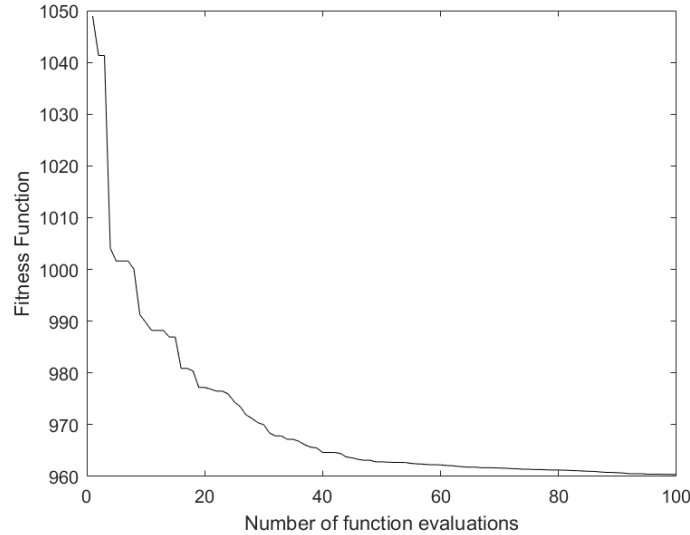


Figure 5.8: Convergence of Case 8 for IEEE30-Bus System

Case 9: Four Objectives: Fuel Cost, Emission, Voltage Deviations and Losses

The aim of this case is to minimize four-conflicted objectives of fuel cost, emission, voltage deviations and losses in the network simultaneously. The objective function is defined by Equation 5.19:

$$f(x, u) = \left(\sum_{i=1}^{NG} a_i + b_i P_{G_i} + c_i P_{G_i}^2 \right) + \lambda_E \times Emission + \lambda_{VD} \times VD + \lambda_P \times P_{Loss} + Penalty \quad (5.19)$$

Substituting related equations, the function will be expressed according to Equation 5.20.

$$f(x, u) = \left(\sum_{i=1}^{NG} a_i + b_i P_{G_i} + c_i P_{G_i}^2 \right) + \lambda_E \times \left(\sum_{k=1}^{NG} (\alpha_i + \beta_i P_{G_i} + \gamma_i P_{G_i}^2) \times 0.01 + \omega_i e^{(\mu_i P_{G_i})} \right) + \lambda_{VD} \times \left(\sum_{P=1}^{NL} |V_{LP} - 1| \right) + \lambda_P \times \left(\sum_{q=1}^m G_{q(ij)} [V_i^2 + V_j^2 - 2V_i V_j - 2V_i V_j \cos(\delta_{ij})] \right) + Penalty \quad (5.20)$$

Where, λ_E , λ_{VD} , and λ_P are the weighting factors, which are selected to balance between the objectives as 19, 21 and 22, respectively [32] [31]. Table 5.5 summarizes the results of implementing the TLBO optimization algorithm by using MATLAB on this case study.

Figure 5.9 illustrates the convergence of fitness function, while all of the four objectives are considered in the multi-objective OPF optimization procedure.

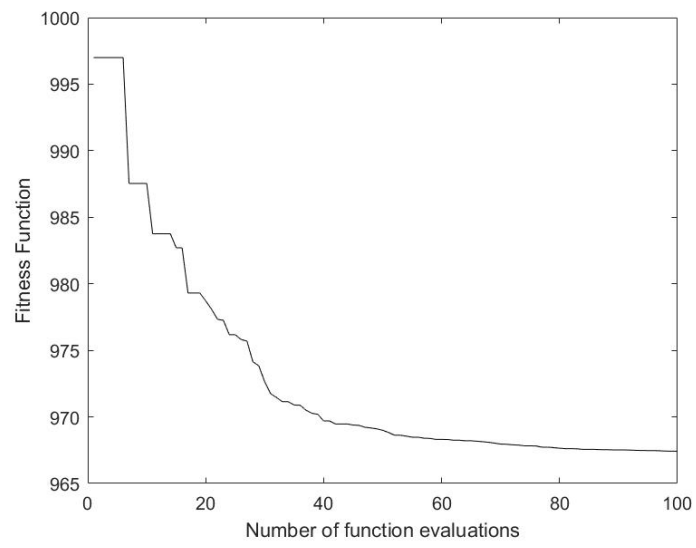


Figure 5.9: Convergence of Case 9 for IEEE 30-Bus System

Table 5.4: Simulation Results of Optimum Solution for IEEE-30 Bus System- Single Objective Cases

Parameters	Min	Max	Case 1	Case 2	Case 3	Case 4
PG2 (MW)	20	80	48.324	67.861	59.5179	79.755
PG5 (MW)	15	50	21.6011	49.9999	50	49.9936
PG8 (MW)	10	35	21.7212	34.9953	20.3846	34.9985
PG11 (MW)	10	30	12.135	29.9999	10.9346	29.9999
PG13 (MW)	12	40	12.0086	39.9996	21.7652	39.9798
VG1 (p.u.)	0.95	1.1	1.0757	0.9865	1.033	1.0595
VG2 (p.u.)	0.95	1.1	1.0541	0.9781	1.0199	1.0545
VG5 (p.u.)	0.95	1.1	1.0244	0.9573	1.0206	1.0365
VG8 (p.u.)	0.95	1.1	1.0317	0.9664	1.0094	1.0444
VG11 (p.u.)	0.95	1.1	1.0991	1.0847	1	1.0676
VG13 (p.u.)	0.95	1.1	1.0758	1.0837	1.0022	1.0691
T11 (p.u.)	0.9	1.1	1.0402	0.9407	0.992	0.9743
T12 (p.u.)	0.9	1.1	0.9347	0.9	0.9019	1.0204
T15 (p.u.)	0.9	1.1	1.0237	0.9599	0.9591	1.0338
T36 (p.u.)	0.9	1.1	0.9663	0.9242	0.9716	0.996
Qc10 (p.u.)	0	5	5	4	4	1
Qc12 (p.u.)	0	5	3	0	4	1
Qc15 (p.u.)	0	5	4	2	3	5
Qc17 (p.u.)	0	5	3	5	2	5
Qc20 (p.u.)	0	5	5	2	5	0
Qc21 (p.u.)	0	5	4	3	3	3
Qc23 (p.u.)	0	5	3	5	3	0
Qc24 (p.u.)	0	5	4	4	3	5
Qc29 (p.u.)	0	5	3	3	5	5
Fuel Cost (\$/h)			800.7832	945.8329	870.632	967.2393
Emission (ton/h)			0.3648	0.2049	0.2644	0.2072
Voltage Deviation (p.u.)			0.9073	0.7326	0.1181	0.6348
Active Power Loss (MW)			9.046	3.6296	6.5758	3.1804

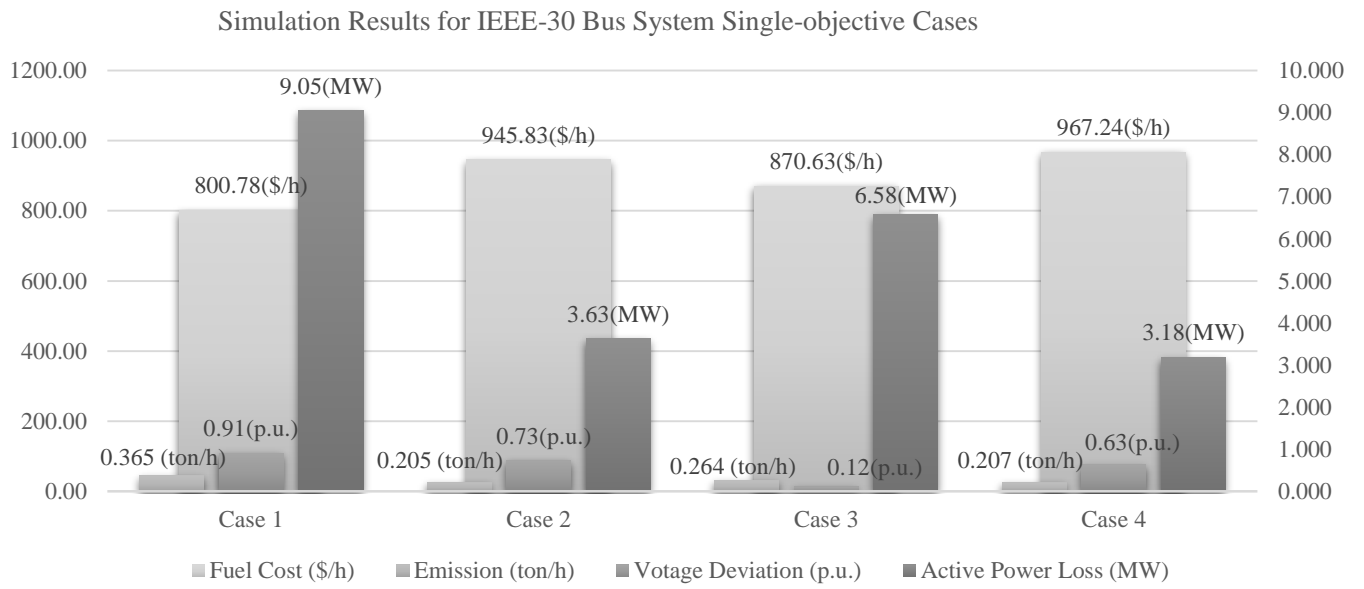


Figure 5.10: Simulation Results of Optimum Solution for IEEE-30 Bus System Single-objective Cases

Table 5.5: Simulation Results of Optimum Solution for IEEE-30 Bus System, Multi-Objective Cases

Parameters	Min	Max	Case 5	Case 7	Case 6	Case 8	Case 9
PG2 (MW)	20	80	58.6876	50.6644	55.6226	52.0053	50.452
PG5 (MW)	15	50	28.0745	21.0879	38.1331	31.9342	31.9004
PG8 (MW)	10	35	34.9809	22.4297	34.9925	34.9939	34.9986
PG11 (MW)	10	30	27.3498	13.3915	29.9965	25.7703	29.5358
PG13 (MW)	12	40	26.6	12.2293	26.8853	21.9452	22.6492
VG1 (p.u.)	0.95	1.1	1.068	1.0397	1.0689	1.0718	1.0534
VG2 (p.u.)	0.95	1.1	1.0545	1.0223	1.0564	1.0555	1.0368
VG5 (p.u.)	0.95	1.1	1.0295	1.0188	1.0347	1.0306	1.0108
VG8 (p.u.)	0.95	1.1	1.0397	1.0049	1.042	1.041	1.02
VG11 (p.u.)	0.95	1.1	1.0532	1.0032	1.0652	1.0876	1.0999
VG13 (p.u.)	0.95	1.1	1.0566	1.0157	1.0679	1.0744	1.0064
T11 (p.u.)	0.9	1.1	0.9933	1.0118	0.9898	0.9985	1.0145
T12 (p.u.)	0.9	1.1	1.0367	0.9007	0.9802	0.9818	1.0249
T15 (p.u.)	0.9	1.1	1.0392	0.9911	1.0117	1.0155	0.9991
T36 (p.u.)	0.9	1.1	0.9836	0.9588	0.9739	0.9705	0.9752
Qc10 (p.u.)	0	5	5	5	4	5	5
Qc12 (p.u.)	0	5	3	4	4	0	5
Qc15 (p.u.)	0	5	5	4	5	3	5
Qc17 (p.u.)	0	5	5	3	2	3	3
Qc20 (p.u.)	0	5	2	5	5	1	3
Qc21 (p.u.)	0	5	5	4	5	0	1
Qc23 (p.u.)	0	5	5	4	3	1	5
Qc24 (p.u.)	0	5	5	2	2	1	2
Qc29 (p.u.)	0	5	1	2	3	2	2
Fuel Cost (\$/h)			836.7029	804.0932	859.5791	830.6375	835.9996
Emission (ton/h)			0.2423	0.3559	0.2286	0.2529	0.2482
Voltage Deviation (p.u.)			0.5471	0.1181	0.8726	0.7155	0.2349
Active Power Loss (MW)			5.4242	9.772	4.5615	5.6193	5.6224

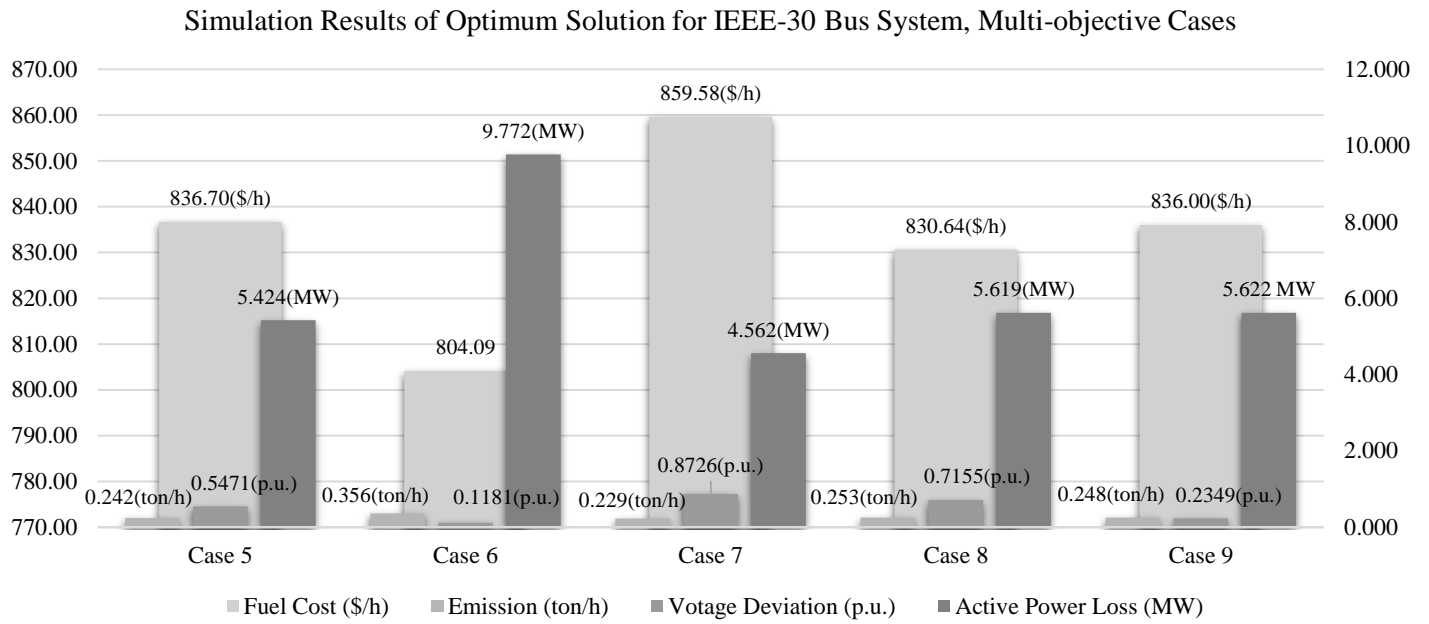


Figure 5.11: Simulation Results of Optimum Solution for IEEE-30 Bus System Multi-objective Cases

5.2.2 IEEE 57-Bus System

The IEEE 57-bus system data [31] is considered to perform a total of 8 case studies for the system. Table 5.6 provides a summary of the system specification. The system active and reactive power demands are 1250.8 MW and 336.4 MVAR. The IEEE 57-bus test system constitutes of 7 generators, 80 transmission line, 17 transformers, and 3 capacitor banks as given in [31]. Upper and lower boundaries of voltage buses and transformer tap settings are between 0.9 & 1.1 p.u. Table 5.7 presents the cost and emission coefficient of generators for IEEE 57-bus system.

Table 5.6: The Main Characteristics of the IEEE 57-Bus Test System [31]

Items	IEEE 57-bus system	
	Quantity	Details
Buses	57	Ref [31]
Branches	80	Ref [31]
Generators	7	Buses:1(Slack), 2, 3, 6, 8 , 9 and 12
Shunt VAR compensation	3	Buses: 18, 25 and 53
Transformer with tab changer	17	Branches: 19, 20, 31, 35, 36, 37, 41, 46, 54, 58, 59, 65, 66, 71, 73, 76, and 80
Control Variables	33	
Connected load		1250.8 MW, 336.4 MVA _r
Load bus voltage range allowed	50	[0.94- 1.06] p.u.

Table 5.7: Cost and Emission Coefficient of Generators for IEEE 57-Bus System [31]

Generator	Bus	a	b	c	d	e	α	β	γ	ω	μ
G1	1	0	20	0.00375	18	0.037	4.091	-5.554	6.49	0.0002	0.286
G2	2	0	40	0.0175	16	0.038	2.543	-6.047	5.638	0.0005	0.333
G3	5	0	20	0.0625	13.5	0.041	6.131	-5.555	5.151	0.000001	0.667
G4	8	0	40	0.00834	18	0.037	3.491	-5.754	6.39	0.0003	0.266
G5	11	0	20	0.025	14	0.04	4.258	-5.094	4.586	0.000001	0.8
G6	13	0	40	0.025	15	0.039	2.754	-5.847	5.238	0.0004	0.288
G7	12	0	20	0.0322581	12	0.045	5.326	-3.555	3.38	0.002	0.2

Case 10: Minimization of Fuel Cost

The basic objective function of this case is to optimize the fuel cost function given by Equation 5.11. The results are provided in Table 5.8, as considered in Case 1. Figure 5.12 shows the minimization of the fuel cost function with respect to the number of evaluations. The optimization process leads to a fuel cost of 41815 \$/hr while satisfying all system constraints.

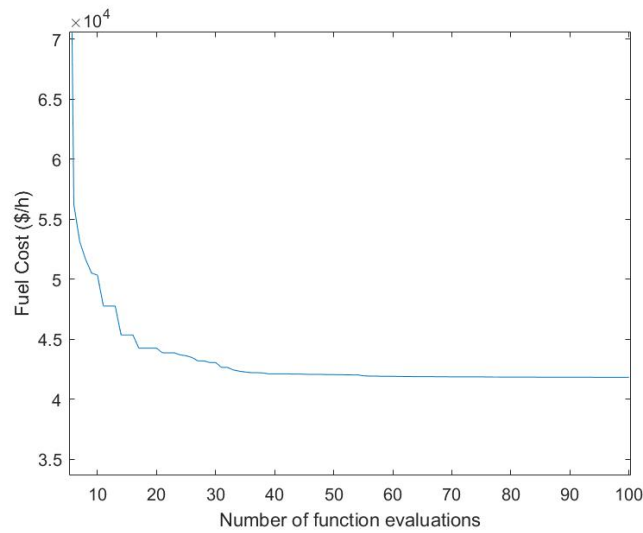


Figure 5.12: Convergence of Case 10 for IEEE 57-Bus System

Case 11: Minimization of Emission

Similar to Case 2, the focus of this case study is on minimizing the total emissions in the OPF optimization problem, defined by Equation 5.12. The results of the optimization procedure by the TLBO algorithm are presented in Table 5.8. In addition, Figure 5.13 shows the minimization of emission function where the optimization procedure provides the best output of 1.0902 ton/hr.

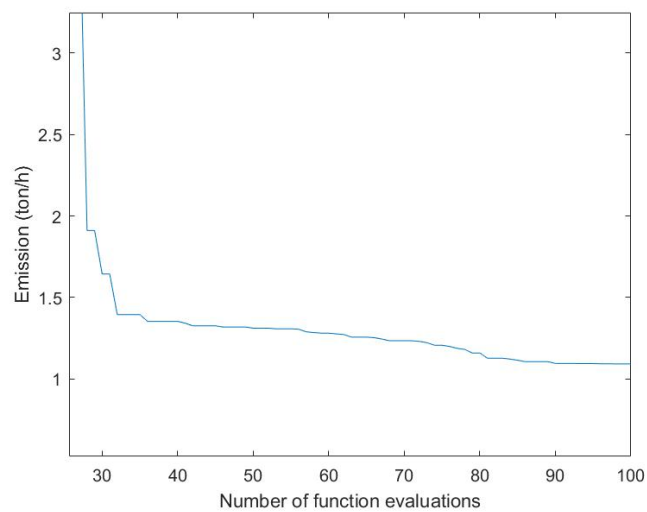


Figure 5.13: Convergence of Case 11 for IEEE 57-Bus System

Case 12: Minimization of Voltage Deviation

This case discusses the same objectives as Case 4. The objective function is given by Equation 5.13. Table 5.8 provides information related to this case. Figure 5.14 illustrates the convergence of function to the optimal solution of 0.7173 p.u after 100 times evaluations by using the TLBO method with MATLAB. Table 5.8 provided more details about the results of implementing this algorithm in this case study.

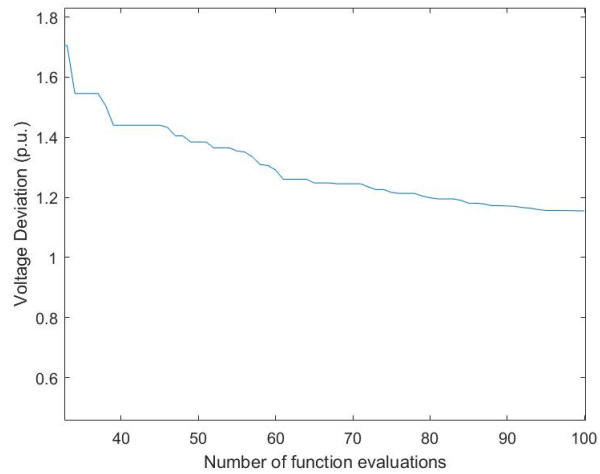


Figure 5.14: Convergence of Case 12 for IEEE 57-Bus System

Case 13: Minimization of Active Power Loss

The aim of this case is to minimize the active power loss in transmission lines, as defined by Equation 5.14. Table 5.7 provides the coefficient and Table 5.8 shows the optimization results. Figure 5.13 shows the optimization procedure results in the real power loss of 13.3375 MW in transmission lines. Table 5.8 provides the details on minimization results.

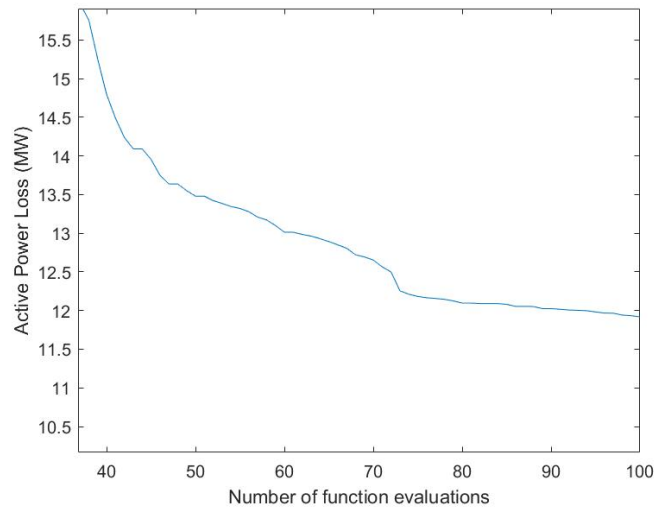


Figure 5.15: Convergence of Case 13 for IEEE 57-Bus System

Case 14: Two Objectives: Minimization of Fuel Cost and Emission

The goal here is to minimize the fuel cost and to improve the emission function of the IEEE 57-test system at the same time. Thus, the objective function is the same as Case 5, given by Equation 5.15 where λ_E is chosen as 1000. Figure 5.16 shows the convergence of fitness function and Table 5.9 presents the optimized solution of this multi-objective optimization problem. For clarity, the convergence of only one objective attaining the best fitness value is included in the fitness diagram for all multi-objective case studies. The best fuel cost value arrives at an increase in cost than Case 10, with final value being 41978 \$/hr and for emission is 1.5153 ton/hr after 100 evaluations.

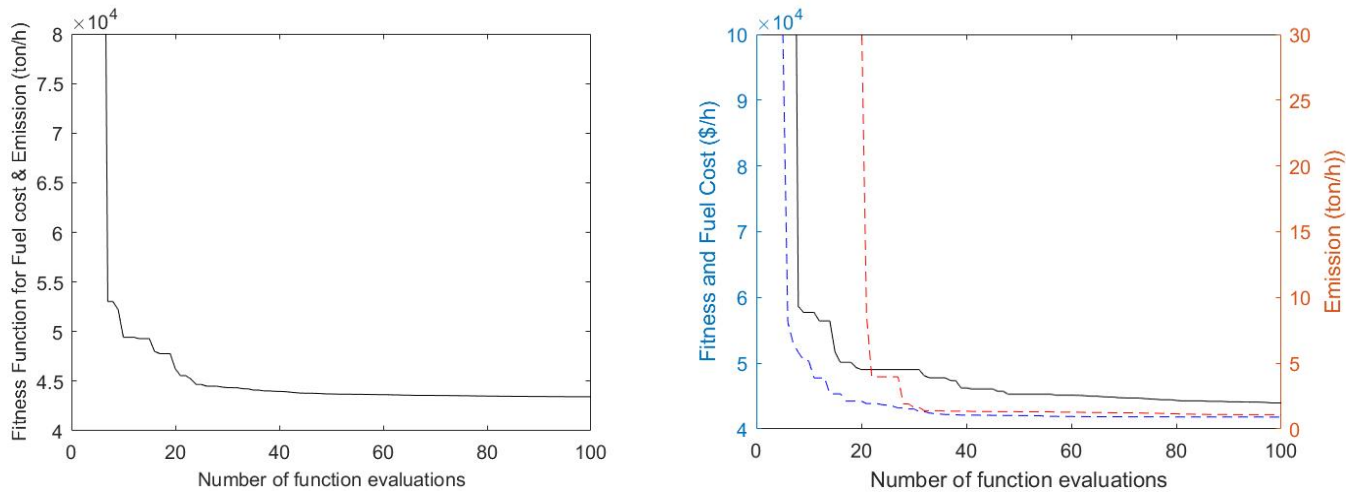


Figure 5.16: Convergence of Case 14 for IEEE 57-Bus System

Case 15: Two Objectives: Minimization of Fuel Cost and Voltage Deviation

As in Case 6, this case is a multi-objective optimization problem. The main purposes of the problem concern minimization of the fuel cost and voltage profile enhancement, as given by Equation 5.16, where λ_{VD} is chosen as 100. Table 5.9 summarizes the results of implementing multi-objective optimization using the TLBO algorithm with MATLAB and Figure 5.17 presents the convergence of fitness function in this case study.

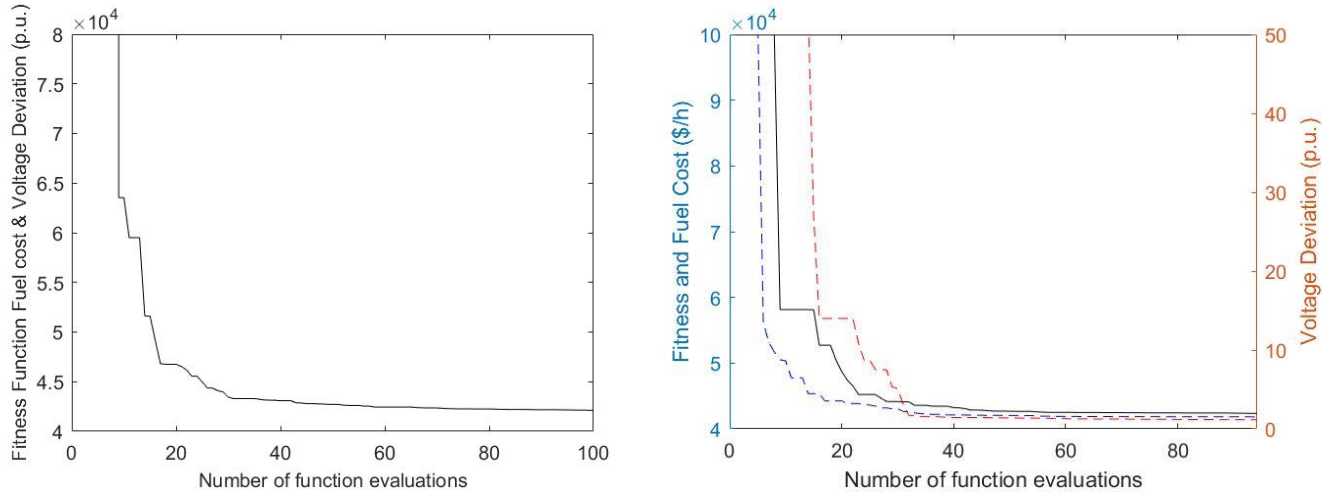


Figure 5.17: Convergence of Case 15 for IEEE 57-Bus System

Case 16: Two Objectives: Minimization of Fuel Cost and Real Power Loss

Similar to Case 7, the goal focuses on the minimization of total fuel cost while monitoring the real power loss. The multi-objective problem is converted to a single function by multiplying weight factors and is used to minimize both objective functions simultaneously according to Equation 5.17. Figure 5.18 shows the convergence of fitness function for a multi-objective optimization of OPF where the fuel cost reaches 41857 \$/hr and real power losses are 15.9728 MW. Table 5.9 summarizes the results of implementing the TLBO algorithm with MATLAB in this case study.

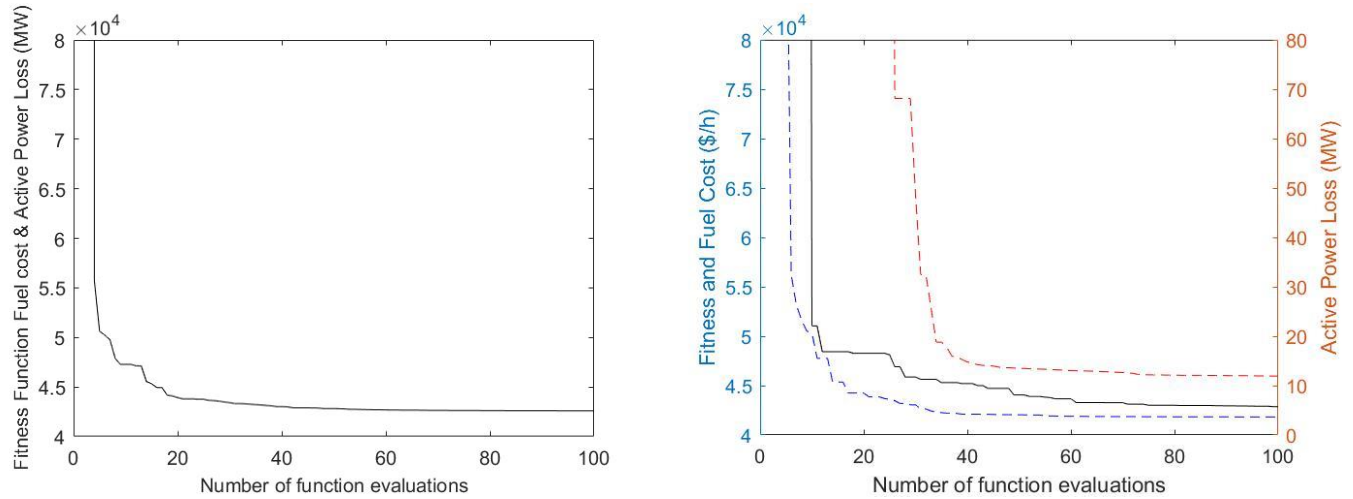


Figure 5.18: Convergence of Case 16 for IEEE 57-Bus System

Case 17: Three Objectives: Minimization of Fuel Cost, Emission, Losses

This case is similar to Case 8. It presents the compromise solution over the optimization of three objective functions consisting of total fuel cost, emission and real power losses at the same time. The problem formulation is given by Equation 5.18.

Figure 5.19 illustrates the convergence of the fitness function and Table 5.9 shows the optimized values with respect to this multi-objective optimization case. The weight factors are selected to balance between the objectives as 19 and 22 for emission and losses, respectively [32]. The fitness function convergence revealed the best values of the function to be 41900 \$/hr, 2.0562 ton/hr, 16.1162MW for fuel cost, emission and losses respectively.

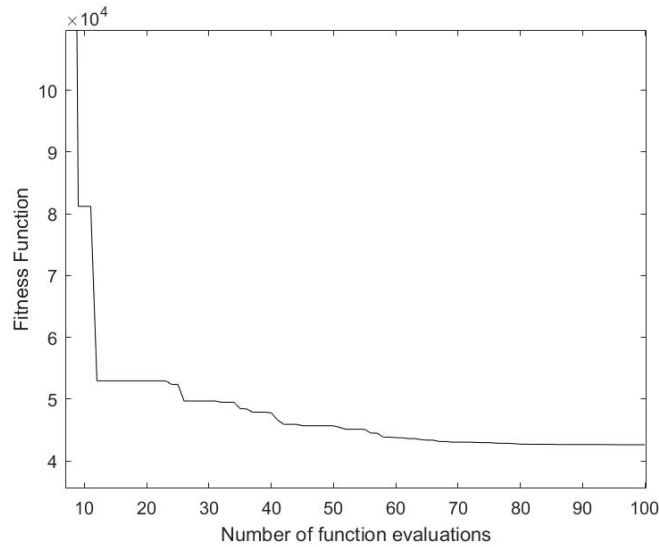


Figure 5.19: Fitness Function Convergence of Case 17 for IEEE 57-Bus System

Case 18: Four Objectives: Fuel Cost, Emission, Voltage Deviations and Losses

As explained in Case 9, the multi-objective case for minimizing fuel cost, emission, and real power losses is converted to single objective function by multiplying a weight factor to the objectives as expressed by Equations 5.19 and 5.20. The weight factors are selected to balance between the objectives as 19, 21 and 22, for emission, voltage deviation, and losses, respectively [32]. Table 5.9 summarizes the results of implementing the TLBO optimization algorithm by using MATLAB in this case study. Figure 5.20 illustrates the convergence of fitness function, while all of the four objectives are considered in the multi-objective optimization procedure.

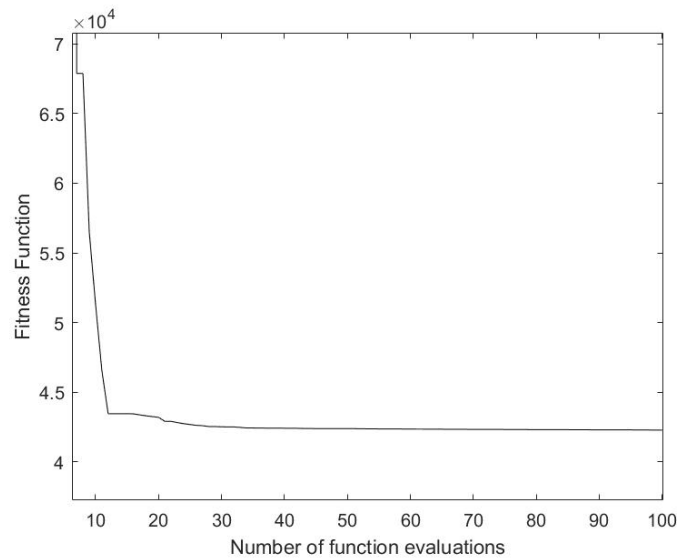


Figure 5.20: Fitness Function Convergence of Case 18 for IEEE 57-Bus System

Table 5.8: Simulation Results of Optimum Solution for IEEE 57-Bus System

Parameters	Min	Max	Case 10	Case 11	Case13	Case 14
PG2 (MW)	30	100	98.1603	99.445	42.3358	10.5523
PG3 (MW)	40	140	49.1818	140	117.2235	82.5886
PG6 (MW)	30	100	69.887	100	47.2586	98.2577
PG8 (MW)	100	550	456.7591	302.6981	281.0804	339.6432
PG9 (MW)	30	100	87.0398	100	59.5569	99.4843
PG12 (MW)	100	410	364.9578	293.3467	302.397	409.1274
V1 (p.u.)	0.95	1.1	0.9852	1.0327	1.0232	0.9665
V2 (p.u.)	0.95	1.1	0.9796	1.0212	1.0081	0.9574
V3 (p.u.)	0.95	1.1	0.9641	1.0073	1.0123	0.9574
V6(p.u.)	0.95	1.1	0.964	0.9887	1.0072	0.9621
V8 (p.u.)	0.95	1.1	0.9775	0.981	1.027	0.9606
V9 (p.u.)	0.95	1.1	0.9514	0.9649	1.0051	0.9511
V12 (p.u.)	0.95	1.1	0.9633	0.9865	1.0219	0.988
Qc18 (p.u.)	0	20	8.48	14.07	7.14	1.6
Qc25 (p.u.)	0	20	22.99	25.46	12.02	19.41
Qc53 (p.u.)	0	20	7.77	0.07	12.05	5.76
T19 (p.u.)	0	20	0.9225	1.0895	1.1	0.9504
T20 (p.u.)	0.9	1.1	0.9025	1.0976	0.9191	0.9002
T31 (p.u.)	0.9	1.1	1.0323	0.9005	1.0151	0.9293
T35(p.u.)	0.9	1.1	1.0288	1.0344	1.0003	0.9597
T36 (p.u.)	0.9	1.1	0.9251	0.9906	1.0395	1.0342
T37 (p.u.)	0.9	1.1	0.9638	0.9443	1.0243	0.9896
T41 (p.u.)	0.9	1.1	0.9431	0.9472	0.9604	0.9006
T46 (p.u.)	0.9	1.1	0.9737	0.9797	0.9183	0.9836
T54 (p.u.)	0.9	1.1	0.9106	0.9	0.9	0.9002
T58 (p.u.)	0.9	1.1	0.9195	0.9255	0.9475	0.9009
T59 (p.u.)	0.9	1.1	0.9152	0.9226	0.9871	0.9094
T65 (p.u.)	0.9	1.1	0.9246	0.9809	0.9887	0.9
T66 (p.u.)	0.9	1.1	0.9088	0.9299	0.9006	0.9
T71 (p.u.)	0.9	1.1	0.9372	0.9299	0.9302	0.9003
T73 (p.u.)	0.9	1.1	0.9985	1.0069	1.0002	0.9002
T76 (p.u.)	0.9	1.1	0.9026	1.0224	0.9787	0.9841
T80 (p.u.)	0.9	1.1	0.956	0.9439	1.0072	0.9
Fuel Cost (\$/h)			41815	45701	50870	43223
Emission (ton/h)			1.8921	1.0902	1.6807	1.5065
Voltage Deviation (p.u.)			1.5192	1.2582	0.7173	1.4022
Active Power Loss (MW)			17.9115	19.816	25.922	13.3375

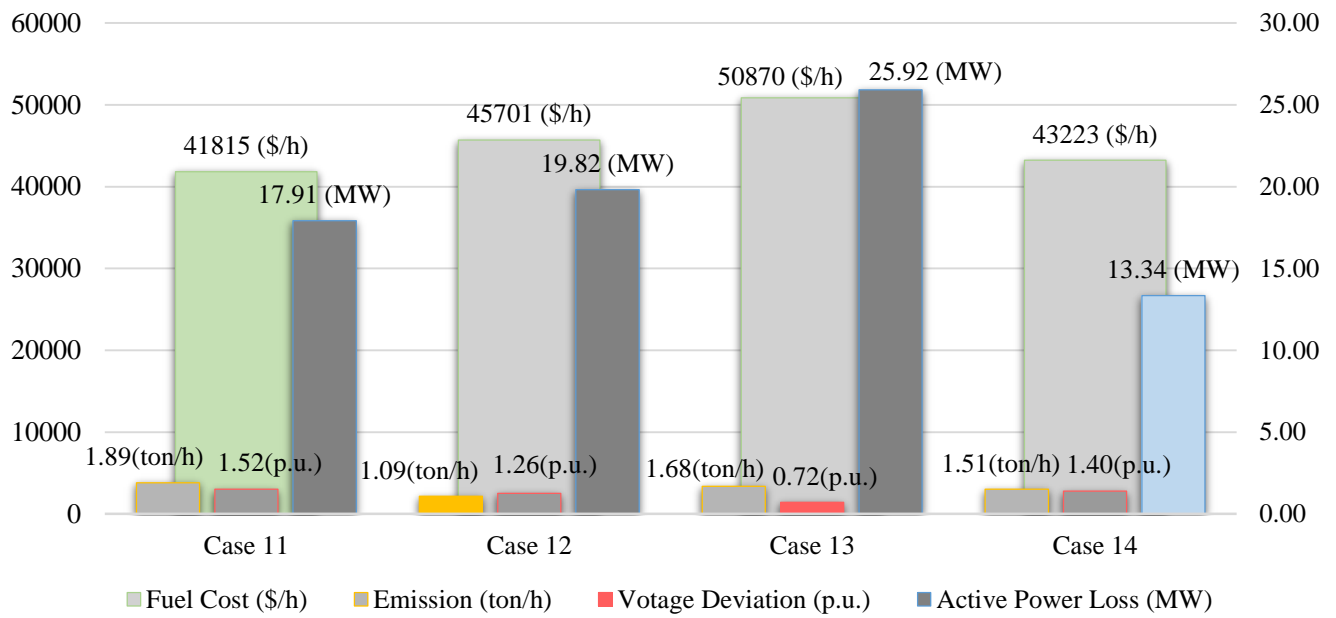


Figure 5.21: Simulation Results of Optimum Solution for IEEE 57-Bus System Single-objective Cases

Table 5.9: Simulation Results of Optimum Solution for IEEE-57 Bus System, Multi-objective Cases

Parameters	Min	Max	Case 14	Case 15	Case 16	Case 17	Case 18
PG2 (MW)	30	100	99.3452	97.6138	81.731	83.8276	66.1226
PG3 (MW)	40	140	51.6008	43.9625	46.9343	49.4303	96.5192
PG6 (MW)	30	100	88.7318	59.4725	64.3148	43.9851	56.9695
PG8 (MW)	100	550	419.6951	464.0246	442.9978	458.6195	455.0092
PG9 (MW)	30	100	97.8437	81.7688	86.7345	78.7001	62.1057
PG12 (MW)	100	410	363.5859	374.0378	399.729	410	357.1104
V1 (p.u.)	0.95	1.1	0.95	0.9869	0.9648	0.9737	0.9992
V2 (p.u.)	0.95	1.1	0.9423	0.9803	0.9598	0.9684	0.9891
V3 (p.u.)	0.95	1.1	0.9405	0.966	0.9509	0.9579	1.0001
V6(p.u.)	0.95	1.1	0.9566	0.9668	0.9508	0.9684	1.0051
V8 (p.u.)	0.95	1.1	0.9781	0.9878	0.9695	0.9802	1.0191
V9 (p.u.)	0.95	1.1	0.9548	0.9639	0.9496	0.9528	0.9881
V12 (p.u.)	0.95	1.1	0.974	0.9837	0.9744	0.966	0.9878
Qc18 (p.u.)	0	20	14.29	6.9	7.46	6.49	5.39
Qc25 (p.u.)	0	20	13.5	12.55	19.23	12	24.57
Qc53 (p.u.)	0	20	19.6	8.06	10.65	9.24	6.45
T19 (p.u.)	0	20	0.9002	0.9001	0.9	0.905	0.9724
T20 (p.u.)	0.9	1.1	0.9072	0.9982	0.9	0.9	1.0532
T31 (p.u.)	0.9	1.1	0.97	0.968	0.9871	0.923	0.9526
T35(p.u.)	0.9	1.1	0.9	1.0152	1.1	0.9076	1.0274
T36 (p.u.)	0.9	1.1	1.0288	0.901	0.9012	0.9	0.907
T37 (p.u.)	0.9	1.1	1.0054	0.997	0.9879	0.9098	0.9341
T41 (p.u.)	0.9	1.1	0.9013	0.9145	0.9	0.9066	0.9311
T46 (p.u.)	0.9	1.1	0.9489	0.9451	0.9188	0.941	0.9941
T54 (p.u.)	0.9	1.1	0.9	0.9452	0.9055	0.9013	0.9998
T58 (p.u.)	0.9	1.1	0.9024	0.9001	0.9126	0.9	0.9121
T59 (p.u.)	0.9	1.1	0.9008	0.913	0.9254	0.9151	0.9712
T65 (p.u.)	0.9	1.1	0.9063	0.9299	0.9074	0.9	0.982
T66 (p.u.)	0.9	1.1	0.9002	0.901	0.9	0.9	0.9629
T71 (p.u.)	0.9	1.1	0.9003	0.9041	0.9031	0.9126	0.9063
T73 (p.u.)	0.9	1.1	1.0267	0.931	0.9857	0.9957	0.9807
T76 (p.u.)	0.9	1.1	0.9048	0.9382	0.9405	0.9	0.9008
T80 (p.u.)	0.9	1.1	0.9	0.9366	0.9013	0.9	0.999
Fuel Cost (\$/h)			41875	41852	41857	41900	42583
Emission (ton/h)			1.5821	1.9748	1.8674	2.0562	1.879
Voltage Deviation (p.u.)			1.5599	1.426	1.5042	1.78	1.5491
Active Power Loss (MW)			17.1518	18.5264	15.9728	16.1162	17.3545
Active Power Loss (p.u.)			17.1518	18.5264	15.9728	16.1162	17.3545

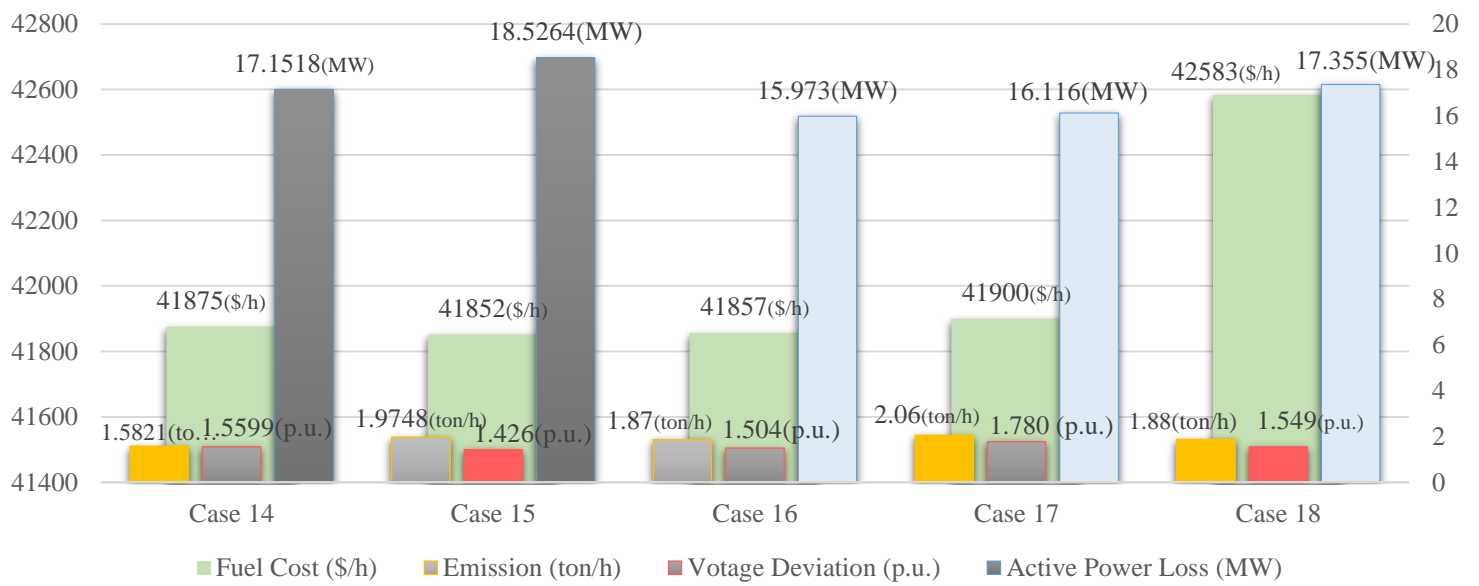


Figure 5.22: Simulation Results of Optimum Solution for IEEE-57 Bus System Multi-objective Cases

5.3 Comparative Study

The results reported in this thesis are compared to some other Artificial Intelligent (AI) methods under the same control variable limits, constraints, and system data for two case studies. It appears from Tables 5.10 and 5.11 that the proposed method outperforms many techniques used to solve different OPF problems. The results are either better or comparable to those obtained by other techniques. This shows the efficiency of the TLBO method for finding the optimum solution.

Table 5.10: Comparison of the Simulation Results for Minimizing Fuel Cost with Different AI Methods
Case 1, IEEE30-Bus System

Method	Cost (\$/hr)	Method Description	Reference
GSA	798.67	Gravitational Search Algorithm	[45]
DSA	799.09	Differential Search Algorithm	[46]
SA	799.45	Simulated Annealing	[47]
DE	799.28	Differential Evolution	[48]
TLBO	800.48	Teaching-Learning-Based Optimization	
FPSO	800.72	Fuzzy Particle Swarm Optimization	[49]
IGA	800.80	Improved Genetic Algorithm	[50]
PSO	800.96	Particle Swarm Optimization	[49]
GAF	801.21	Fuzzy Genetic Algorithm	[49]
ICA	801.84	Imperialistic Competitive Algorithm	[51]
EGA	801.06	Enhanced Genetic Algorithm	[52]
TS	802.29	Tabu Search	[53]
MDE	802.37	Modified Differential Evolutionary Algorithm	[54]
IEP	802.46	Improved Evolutionary Programming	[55]
EP	802.62	Evolutionary Programming	[56]
RGA	804.02	Refined Genetic Algorithm	[57]
GM	804.85	Gradient Method	[58]
GA	805.94	Genetic Algorithm	[57]

Table 5.11: Comparison of the Simulation Results in Multi-Objective Optimization of Fuel Cost and Voltage Deviation, Case 5, IEEE30-Bus System

Method	Cost (\$/hr)	Voltage Deviation(p.u.)	Objective Function	Method Description	Reference
TLBO	803.8268	0.09750	813.5786	Teaching-Learning-Based Optimization	
DSA	803.8274	0.09770	813.5961	Differential Search Algorithm	[46]
GSA	804.3148	0.09330	813.6417	Gravitational Search Algorithm	[45]
BBO	804.9982	0.10400	815.1982	Biogeography-Base Optimization	[59]
PSO	806.3800	0.08910	815.2900	Particle Swarm Optimization	[49]
EM	804.2600	0.12700	816.9600	Electromagnetism-Like Mechanism	[60]
DE	805.2619	0.13670	818.8319	Differential Evolution	[48]

5.4 Conclusions

This chapter has presented the application of multi-objective optimization in solving the Optimal Power Flow(OPF) problem. The objective functions are: fuel cost function, emission, voltage deviation, and transmission losses. The Teaching-Learning-Based Optimization (TLBO) algorithm has been successfully and effectively implemented to the IEEE 30-bus and IEEE 57-bus power system for single and multi-objective cases to reflect the performance of the power system. Fitness function gradually reduces in all the minimization cases. It is worthwhile to mention that unlike the other optimization algorithm, TLBO does not require any internal parameter to tune.

Evidently, the OPF fitness function gradually reduces in all optimization cases, despite some irregularities occurring during the process of convergence due to the non-linear relationship of the objective functions with control parameters of the network. A comparative study is provided to highlight the efficiency of the algorithm in comparison with other artificial intelligence methods.

Chapter 6

6. Conclusion and Future Work

The primary purpose of this thesis is to present the general concepts and classifications of multi-objective optimization methods and draw the focus to its applications on power systems.

The optimal power flow problem is a non-linear, constrained optimization problem with several objective functions. Multi-objective optimization has been implemented to optimize the OPF problem for different objective functions considering fuel cost, emission, voltage deviation, and power losses.

The Teaching-Learning-Based Optimization (TLBO) algorithm is a popular method among metaheuristic techniques with relatively competitive performances. It is inspired by passing the knowledge from teacher to learners or by the interaction between the learners themselves. TLBO is employed to optimize the OPF problem by using MATLAB for single and multi-objective OPF case studies. Simulation results and comparisons highlight the effectiveness of the method for solving multi-objective OPF problems and reflect the performance of the power system. This thesis has considered four key problems for power systems: economy, environmental emission, voltage deviation, and transmission losses. The outcomes of the presented research reflect the effectiveness of approaches and potential benefits for power system planning and operation.

6.1 Summary of the Research and Contribution of Thesis

- Application of multi-objective optimization in solving the optimal power flow problem has been presented.
- The Teaching-Learning-Based Optimization (TLBO) algorithm has been successfully and effectively implemented to standard case studies for single and multi-objective cases.
- Unlike other optimization algorithms, TLBO does not require any internal parameter to tune.
- The OPF fitness function for all presented case studies gradually reduces using TLBO method, despite some irregularities in the process of convergence, which arises due to the nonlinear relationship between the objective functions and control parameters of the network.
- The efficiency of TLBO method is shown with a comparative study with other intelligent methods for a single objective and a multi-objective case study in this thesis.
- The ability of TLBO to provide the global optimum solutions in comparatively less number of function evaluations can be studied in OPF for future works.

6.2 Recommendations of Future Work

Some of the potential research areas for improving the performance of the algorithm in the future is presented in this section.

- The concepts of learning through tutorials and self-motivated leaning can be studied in future research.
- Using the adaptive teaching factor for TLBO is a new discussion for improving the performance of the algorithm. The adaptive teaching factor allows the students to acquire partial knowledge from the instructor where as in the basic TLBO the students could learn either everything or nothing from the presented topic [61].
- Number of teachers on the fitness value of the objective function could be increased to effect the efficiency of the algorithm.
- TLBO with a variable-population scheme can be studies for optimization problems.

- Experimentation of TLBO on very large-scale systems is another interesting area for future studies in order to show the capability and efficiency of the system for handling a system with large number of variables and constraints.
- Using an adaptive penalty factor for system constraints in solving multi-objective optimal

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